Spatially Controlled Relay Beamforming

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***Abstract*—We consider the problem of enhancing Quality-of-Service (QoS) in mobile relay beamforming networks, by optimally controlling relay motion, at the presence of a dynamic channel. We assume a time slotted system, where the relays update their positions before the beginning of each time slot. Modeling the wireless channel as a Gaussian spatiotemporal stochastic field, we propose a novel 2-stage stochastic programming approach for optimally specifying relay positions and beamforming weights, such that the expected QoS of the network is maximized, based on causal Channel State Information (CSI) and under a total relay transmission power budget. This results in a scheme where, at each time slot, apart from optimally beamforming to the destination, the relays also optimally decide their positions at the next time slot, based on their causal experience. The stochastic program considered is shown to be equivalent to a set of simple subproblems, which may be solved in a naturally distributed fashion, one at each relay. However, exact evaluation of the objective of each subproblem is impossible. To mitigate this issue, we propose three efficient, theoretically grounded surrogates to the original subproblems, which rely on the Sample Average Approximation method, the Gauss-Hermite Quadrature, and the Method of Statistical Differentials, respectively. The efficacy and several interesting properties of the proposed approach are demonstrated via numerical simulations. In particular, we report a substantial improvement of about 80% on the average network SINR at steady state, compared to randomized relay motion. This shows that strategic relay motion control can result in substantial performance gains, as far as QoS maximization is concerned.**

***Index Terms*—Spatially Controlled Relay Beamforming, Network Mobility Control, Network Utility Optimization, Distributed Cooperative Networks, Stochastic Programming.**

I. INTRODUCTION

DIstributed, networked communication systems, such as relay beamforming networks [3]–[9] (e.g., Amplify & Forward (AF)) are typically designed without explicitly considering how the positions of the networking nodes might affect communication quality. However, communication is dependent on node positioning and, further, in most practical settings, the Channel State Information (CSI) observed by each node, per channel use, is both spatially and temporally correlated. It is, therefore, reasonable to ask if and how system performance could be improved by controlling the positions of certain network nodes, exploiting the spatiotemporal dependencies of the wireless medium.

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Autonomous node mobility has been proposed as effective means of enhancing performance in various distributed networking settings. In [10], optimal transmit AF beamforming has been combined with potential field based relay mobility control in multiuser networks, minimizing relay transmit power, while respecting certain Quality-of-Service (QoS) constraints. In [11], decentralized jammer motion control has been jointly combined with noise nulling and cooperative jamming, for maximizing network secrecy rate. In [12], optimal relay positioning has been studied in systems where multiple relays deliver information to a destination, at the presence of an eavesdropper, with a goal of maximizing or achieving a target level of ergodic secrecy. In the complementary context of communication aware (comm-aware) robotics, node mobility has been exploited in distributed robotic networks for maintaining reliable, in-network communication connectivity [13]–[17], and optimizing network energy management [18]. Networked node motion control has also been exploited in special purpose applications, such as networked robotic surveillance [19] and target tracking [20].

In [10]–[12], the links among network nodes (or related statistics) are assumed to be available in the form of static channel maps, during the whole motion of the jammers/relays. However, this is oversimplifying in scenarios where the channels change significantly in time and space [21]–[23]. Most recently, in [24], unmanned vehicle motion control was considered for minimizing energy requirements in a transmit (uplink) beamforming scenario, assuming a commonly employed, spatially varying "log-normal" channel model [23].

In this paper, we consider the problem of optimally and dynamically updating relay positions in single source/destination *relay* beamforming networks, in a fully dynamic, *space-time varying* channel environment. Different from [10]–[12], we model the wireless channel as a *spatiotemporal stochastic field*; this approach may be seen as a versatile extension of the channel model of [23]. We then propose a 2-*stage stochastic programming* approach, optimally specifying relay positions and beamforming weights, such that the *expected Signal-to-Interference+Noise Ratio (SINR)* or *QoS* at the destination is maximized, on the basis of *causal CSI*, and subject to a *total power constraint at the relays*. At each time slot, the relays not only beamform to the destination, but also optimally *predictively* decide their positions at the next time slot, based on their experience *so far*. This novel, cyber-physical system approach to relay beamforming is termed here as *Spatially Controlled Relay Beamforming*.

First, it is shown that the proposed 2-stage problem is equivalent to a set of two dimensional subproblems, which can be solved in a *distributed fashion*, one at each relay, *without the need for intermediate exchange of messages* among the relays. However, the objective of each subproblem turns out to be impossible to evaluate analytically. In order to overcome this difficulty, we propose *three theoretically sound and numerically efficient* approximations to each subproblem, each acting as a *surrogate* to the former. Each of the proposed surrogates relies on the so-called *Sample Average Approximation (SAA)* method [25], the *Gauss-Hermite Quadrature (GHQ)* [26], and the *Method of Statistical Differentials* [27], respectively, and features a natural trade-off between performance and accuracy. If applicable, rigorous theoretical analysis is also presented.

The efficacy of the proposed approach is confirmed via extensive numerical simulations. In particular, we reveal an interesting and rather useful feature of the proposed system: Although we optimize a 1*-step lookahead objective* at each time slot (i.e., our formulation is *myopic*), the *expected network QoS exhibits an increasing trend across time slots*, under optimal decision making at the relays. Most importantly, we experimentally report *an average improvement of about* 80% *on the average network SINR at steady state*, compared to purely randomized relay motion, showing that strategic relay mobility can result in substantial performance gains, as far as enhancement of QoS is concerned. The resilience of the proposed approach against random motion failures in the network is also experimentally studied; more specifically, it is demonstrated that, while in steady state, motion failures do not result in significant system deterioration, as long as the temporal correlation of the channel is sufficiently strong.

An example of a real-world application of our problem setting may be found in [17], where multiple robotic routers, noncollaboratively relaying information between a source and a destination in a multihop fashion, are dynamically controlled in space, so that either the end-to-end Bit Error Rate (BER) or power requirements of the system are optimized. Our problem is similar, in the sense that we also consider maximization of QoS between a source and a destination. However, in our setting, and different from [17], the mobile relays *collaboratively beamform* to the destination in a 2*-hop*, AF fashion. The framework proposed in [17] has been experimentally confirmed (also in [17]), and the channel model assumed in [17] constitutes a *reduced, temporally static* version of the time varying model considered herein. Thus, our channel model includes that of [17] as a special case, and our approach could be readily applied to the experimental setting of [17], as well.

More generally, spatially controlled relay beamforming finds relevance in several important application domains. Let us present two examples. The first one is *improving network QoS in urban and suburban environments* (e.g., a big city), where the relays would be moving either on the ground, or in the air, but at relatively low altitude. Particular applications include search-and-rescue missions, surveillance, high quality environmental sampling, reconfigurable and infrastructureless sensor networks, on-demand industrial monitoring, wide area WiFi, and physical layer security. Another application domain of interest is *enabling reliable communications in the area of a battle field*. In this case, wireless relays would be dynamically deployed in space (either ground or air), in order to improve jammed communications, compensate for destroyed, or aid existing infrastructure, or even optimally intercept enemy communications. Common characteristic of the aforementioned applications is potential mobility of the respective communication endpoints. In such cases, *wireless* networking is essential, and this paper provides a novel methodological approach for substantially improving communication quality by introducing and exploiting mobility of the involved relaying networking nodes. For further details on feasibility, usefulness and necessity of spatially controlled *WiFi* communications, the reader is also referred to the recent experimental study [28].

The paper is organized as follows. Sections II and III introduce the networking model and the spatiotemporal channel model under study, respectively. The scheduling protocol of the spatially controlled system, and our problem formulation are presented in clear order in Section IV. Our approximate solution methodology is developed in Section V, along with relevant theoretical justification. Section VI briefly discusses computational issues of the proposed approach. In Section VII, we present our numerical simulations, along with the relevant discussion. Finally, Section VIII concludes the paper.

Notation: Matrices and vectors will be denoted by boldface uppercase and boldface lowercase letters, respectively. Calligraphic letters and formal script letters will denote sets and σ-algebras, respectively. The operators $(\cdot)^*$, $(\cdot)^{\boldsymbol{T}}$ and $(\cdot)^{\boldsymbol{H}}$ will denote conjugation, transposition and conjugate transposition, respectively. The ℓ_p-norm of $\boldsymbol{x} \in \mathbb{R}^n$ is $\|\boldsymbol{x}\|_p \triangleq (\sum_{i=1}^n |x(i)|^p)^{1/p}$, for $\mathbb{N} \ni p \geq 1$. For $\mathbb{N} \ni N \geq 1$, $\mathbb{S}^N$, $\mathbb{S}^N_{+(+)}$ will denote the sets of symmetric and symmetric positive (semidefinite) matrices, respectively. The N-dimensional identity operator will be denoted as $\mathbf{I}_N$. Also, we define $\mathfrak{J} \triangleq \sqrt{-1}$, $\mathbb{N}^+ \triangleq \{1, 2, \ldots\}$, $\mathbb{N}^+_n \triangleq \{1, 2, \ldots, n\}$, $\mathbb{N}_n \triangleq \{0\} \cup \mathbb{N}^+_n$ and $\mathbb{N}^m_n \triangleq \mathbb{N}^+_n \setminus \mathbb{N}^+_{m-1}$, for positive naturals $n > m$.

II. SYSTEM MODEL

On a compact, square planar region $\mathcal{W} \subset \mathbb{R}^2$, we consider a wireless cooperative network consisting of one source, one destination and $R \in \mathbb{N}^+$ assistive relays, as shown in Fig. 1. Each entity of the network is equipped with a single antenna, being able for both information reception and broadcasting/transmission. The source and destination are stationary (for simplicity) and located at $\mathbf{p}_S \in \mathcal{W}$ and $\mathbf{p}_D \in \mathcal{W}$, respectively, whereas the relays are assumed to be mobile; each relay $i \in \mathbb{N}^+_R$ moves along a trajectory $\mathbf{p}_i(t) \in \mathcal{S} \subset \mathcal{W} - \{\mathbf{p}_S, \mathbf{p}_D\} \subset \mathcal{W}$, where, in general, $t \in \mathbb{R}_+$, and where $\mathcal{S}$ is a finite set. We also define the supervector $\mathbf{p}(t) \triangleq \left[\mathbf{p}_1^{\boldsymbol{T}}(t)\ \mathbf{p}_2^{\boldsymbol{T}}(t) \ldots \mathbf{p}_R^{\boldsymbol{T}}(t)\right]^{\boldsymbol{T}} \in \mathcal{S}^R \subset \mathbb{R}^{2R \times 1}$. Additionally, we assume that the relays can cooperate with each other, either by exchanging local messages, or by communicating with a local fusion center, through a dedicated channel. Hereafter, all probabilistic arguments made below presume the existence of a complete base probability space of otherwise arbitrary structure, defined by a triplet $(\Omega, \mathscr{F}, \mathcal{P})$.

Assuming that a direct link between the source and destination does not exist, the relays are assistive to the communication, operating in a classical, two phase AF relaying mode. Fix a $T > 0$, and *divide the time interval* $[0, T]$ *into* N_T *time slots*, with $t \in \mathbb{N}^+_{N_T}$ denoting the respective time slot. Let $s(t) \in \mathbb{C}$, with $\mathbb{E}\left\{|s(t)|^2\right\} \equiv 1$, denote the symbol to be transmitted at time slot t. Also, assuming a flat fading channel model, as

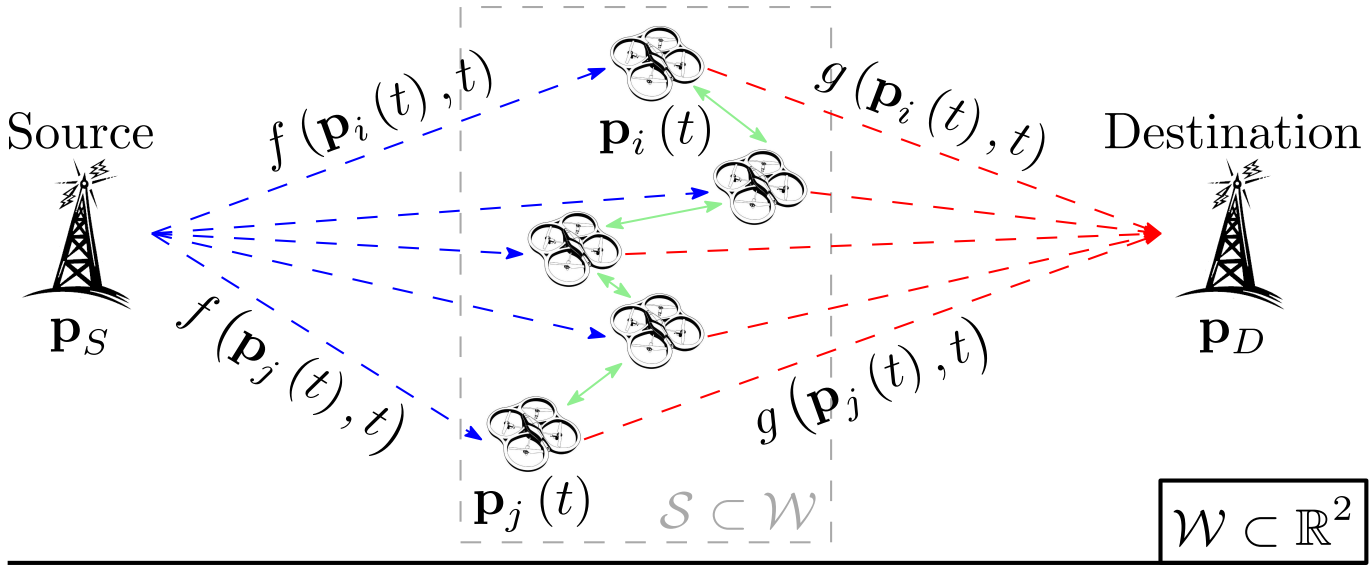

Figure 1: A concept schematic of the system model considered.

well as channel reciprocity and quasistaticity in each time slot, let the sets $\{f_i(t) \in \mathbb{C}\}_{i\in\mathbb{N}_R^+}$ and $\{g_i(t) \in \mathbb{C}\}_{i\in\mathbb{N}_R^+}$ contain the *random, spatiotemporally varying* source-relay and relay-destination channel gains, respectively. These are further assumed to be *realizations* of the *random channel fields* or *maps* $f(\mathbf{p}, t)$ and $g(\mathbf{p}, t)$, respectively, that is, $f_i(t) \equiv f(\mathbf{p}_i(t), t)$ and $g_i(t) \equiv g(\mathbf{p}_i(t), t)$, for all $i \in \mathbb{N}_R^+$ and for all $t \in \mathbb{N}_{N_T}^+$. Then, if $P_0 > 0$ denotes the transmission power of the source, during AF phase 1, the signals received at the relays can be expressed as $r_i(t) \triangleq \sqrt{P_0} f_i(t)\, s(t) + n_i(t) \in \mathbb{C}$, for all $i \in \mathbb{N}_R^+$ and for all $t \in \mathbb{N}_{N_T}^+$, where $n_i(t) \in \mathbb{C}$, with $\mathbb{E}\left\{|n_i(t)|^2\right\} \equiv \sigma^2$, constitutes a zero mean noise process at the i-th relay, independent across relays. During AF phase 2, all relays modulate their received signal by a weight $w_i(t) \in \mathbb{C}, i \in \mathbb{N}_R^+$, and simultaneously retransmit it. The signal received at the destination is $y(t) \triangleq \sum_{i\in\mathbb{N}_R^+} w_i(t)\, g_i(t)\, r_i(t) + n_D(t)$, and can be further expressed as

$$y(t) \equiv \underbrace{\sqrt{P_0} \sum_{i\in\mathbb{N}_R^+} w_i(t)\, g_i(t)\, f_i(t)\, s(t)}_{\text{signal (transformed)}} + \underbrace{\sum_{i\in\mathbb{N}_R^+} w_i(t)\, g_i(t)\, n_i(t) + n_D(t)}_{\text{interference + reception noise}} \in \mathbb{C}, \quad (1)$$

for all $t \in \mathbb{N}_{N_T}^+$, where $n_D(t) \in \mathbb{C}$, with $\mathbb{E}\left\{|n_D(t)|^2\right\} \equiv \sigma_D^2$, constitutes a zero mean, white noise process at the destination.

In the following, it is assumed that the channel fields $f(\mathbf{p}, t)$ and $g(\mathbf{p}, t)$ may be *statistically dependent both spatially and temporally*, and that, *as usual*, the processes $s(t)$, $[f(\mathbf{p}, t)\ g(\mathbf{p}, t)]$, $n_i(t)$ for all $i \in \mathbb{N}_R^+$, and $n_D(t)$ are mutually independent. Also, we will assume that, at each time slot t, CSI $\{f_i(t)\}_{i\in\mathbb{N}_R^+}$ and $\{g_i(t)\}_{i\in\mathbb{N}_R^+}$ is known *exactly* to all relays. This may be achieved through pilot based estimation.

Remark 1. The assumption that the source and destination are stationary is made for technical simplicity. In fact, both source and destination *are allowed to be mobile*, as long as their trajectories are known to the relays across all time slots, *one slot ahead in the future*. In other words, source and destination are required to communicate, at time slot t, their future positions at time slot $t+1$, to the relays. Additionally, as also assumed for the relays as well (see Section IV.A), in case source and destination are mobile, they are both assumed not to communicate and move simultaneously. ■

III. Spatiotemporal Wireless Channel Modeling

This section introduces and discusses a stochastic model for describing a spatiotemporally varying wireless channel.

A. Gaussian Field Channel Modeling in the dB Domain

At each space-time point $(\mathbf{p}, t) \in \mathcal{S} \times \mathbb{N}_{N_T}^+$, the source-relay channel field may be decomposed as the product of three space-time varying components [29], as

$$f(\mathbf{p}, t) \equiv \underbrace{f^{PL}(\mathbf{p})}_{\text{path loss}} \underbrace{f^{SH}(\mathbf{p}, t)}_{\text{shadowing}} \underbrace{f^{MF}(\mathbf{p}, t)}_{\text{fading}} e^{\jmath \frac{2\pi \|\mathbf{p}-\mathbf{p}_S\|_2}{\lambda}}, \quad (2)$$

where $\lambda > 0$ denotes the communication wavelength, $f^{PL}(\mathbf{p}) \triangleq \|\mathbf{p} - \mathbf{p}_S\|_2^{-\ell/2}$ is the *path loss field*, with $\ell > 0$ being the path loss exponent, $f^{SH}(\mathbf{p}, t) \in \mathbb{R}$ is the *shadowing field*, and $f^{MF}(\mathbf{p}, t) \in \mathbb{C}$ is the *multipath fading field*.

The same decomposition holds in direct correspondence for the relay-destination channel field, $g(\mathbf{p}, t)$. It is assumed that the (vector) fields $f^{MF}(\mathbf{p}, t)$ and $\left[f^{MF}(\mathbf{p}, t)\ g^{MF}(\mathbf{p}, t)\right]$ are independent of $g^{MF}(\mathbf{p}, t)$ and $\left[f^{SH}(\mathbf{p}, t)\ g^{SH}(\mathbf{p}, t)\right]$, respectively. It is further assumed that the phase field of $f^{MF}(\mathbf{p}, t)$ is independent of the magnitude field $\left|f^{MF}(\mathbf{p}, t)\right|$. It then follows that the vector fields $\left[\left|f^{MF}(\mathbf{p}, t)\right|\ \left|g^{MF}(\mathbf{p}, t)\right|\right]$ and $\left[f^{SH}(\mathbf{p}, t)\ g^{SH}(\mathbf{p}, t)\right]$ are independent, as well.

We are interested in the magnitudes of both fields $f(\mathbf{p}, t)$ and $g(\mathbf{p}, t)$. Instead of working with (2), it is more preferable to work in logarithmic scale. We may define the *log-scale magnitude field*

$$F(\mathbf{p},t) \triangleq \alpha_S(\mathbf{p})\,\ell + \sigma_S(\mathbf{p},t) + \xi_S(\mathbf{p},t), \tag{3}$$

where we identify

$$-\alpha_S(\mathbf{p}) \triangleq 10\log_{10}(\|\mathbf{p}-\mathbf{p}_S\|_2), \tag{4}$$

$$\sigma_S(\mathbf{p},t) \triangleq 10\log_{10}\left(f^{SH}(\mathbf{p},t)\right)^2 \quad \text{and} \tag{5}$$

$$\xi_S(\mathbf{p},t) \triangleq 10\log_{10}\left|f^{MF}(\mathbf{p},t)\right|^2 - \rho, \quad \text{with} \tag{6}$$

$$\rho \triangleq \mathbb{E}\left\{10\log_{10}\left|f^{MF}(\mathbf{p},t)\right|^2\right\}, \tag{7}$$

for all $(\mathbf{p},t) \in \mathcal{S}\times\mathbb{N}^+_{N_T}$. It is trivial to show that the magnitude of $f(\mathbf{p},t)$ may be reconstructed via the *bijective* formula

$$|f(\mathbf{p},t)| \equiv 10^{\rho/20}\exp\left((\log(10)/20)\,F(\mathbf{p},t)\right), \tag{8}$$

for all $(\mathbf{p},t) \in \mathcal{S}\times\mathbb{N}^+_{N_T}$. Regarding $g(\mathbf{p},t)$, the log-scale field $G(\mathbf{p},t)$ is defined in the same fashion, replacing "S" by "D".

For each relay $i \in \mathbb{N}^+_R$, let us define the respective log-scale channel magnitude processes $F_i(t) \triangleq F(\mathbf{p}_i(t),t)$ and $G_i(t) \triangleq G(\mathbf{p}_i(t),t)$, $t \in \mathbb{N}^+_{N_T}$. Of course, we may stack all the $F_i(t)$'s, resulting in the vector additive model

$$\boldsymbol{F}(t) \triangleq \boldsymbol{\alpha}_S(\mathbf{p}(t))\,\ell + \boldsymbol{\sigma}_S(t) + \boldsymbol{\xi}_S(t) \in \mathbb{R}^{R\times 1}, \tag{9}$$

where $\boldsymbol{\alpha}_S(t)$, $\boldsymbol{\sigma}_S(t)$ and $\boldsymbol{\xi}_S(t)$ are defined accordingly. We can also define $\boldsymbol{G}(t) \triangleq \boldsymbol{\alpha}_D(\mathbf{p}(t))\,\ell + \boldsymbol{\sigma}_D(t) + \boldsymbol{\xi}_D(t) \in \mathbb{R}^{R\times 1}$, with each quantity in direct correspondence with (9). In the same manner, the log-scale shadowing and multipath fading processes are defined as $\sigma^i_{S(D)}(t) \triangleq \sigma_{S(D)}(\mathbf{p}_i(t),t)$ and $\xi^i_{S(D)}(t) \triangleq \xi_{S(D)}(\mathbf{p}_i(t),t)$, $t \in \mathbb{N}^+_{N_T}$, respectively.

Next, let us focus on the joint spatiotemporal dynamics of $\{|f_i(t)|\}_i$ and $\{|g_i(t)|\}_i$, modeled through those of the shadowing components of $\{F_i(t)\}_i$ and $\{G_i(t)\}_i$. It is assumed that, for any N_T and any *deterministic* ensemble of positions of the relays in $\mathbb{N}^+_{N_T}$, say $\{\mathbf{p}(t)\}_{t\in\mathbb{N}^+_{N_T}}$, the random vector

$$\left[\boldsymbol{F}^{\boldsymbol{T}}(1)\;\boldsymbol{G}^{\boldsymbol{T}}(1)\;\ldots\;\boldsymbol{F}^{\boldsymbol{T}}(N_T)\;\boldsymbol{G}^{\boldsymbol{T}}(N_T)\right]^{\boldsymbol{T}} \in \mathbb{R}^{2RN_T\times 1} \tag{10}$$

is *jointly Gaussian* with known mean and covariance matrix [23], [30]. More specifically, on a per node basis, we let $\xi^i_{S(D)}(t) \overset{i.i.d.}{\sim} \mathcal{N}\left(0,\sigma^2_\xi\right)$ and $\sigma^i_{S(D)}(t) \overset{i.d.}{\sim} \mathcal{N}\left(0,\eta^2\right)$, for all $t \in \mathbb{N}^+_{N_T}$ and $i \in \mathbb{N}^+_R$ [23], [31]. In particular, extending Gudmundson's model [32], we propose defining the spatiotemporal correlations of the shadowing part of the channel as

$$\mathbb{E}\left\{\sigma^i_S(k)\,\sigma^j_S(l)\right\} \triangleq \eta^2 e^{-\frac{\|\mathbf{p}_i(k)-\mathbf{p}_j(l)\|_2}{\beta}-\frac{|k-l|}{\gamma}}, \tag{11}$$

and correspondingly for $\left\{\sigma^i_D(t)\right\}_{i\in\mathbb{N}^+_R}$, and additionally,

$$\mathbb{E}\left\{\sigma^i_S(k)\,\sigma^j_D(l)\right\} \triangleq \mathbb{E}\left\{\sigma^i_S(k)\,\sigma^j_S(l)\right\} e^{-\frac{\|\mathbf{p}_S-\mathbf{p}_D\|_2}{\delta}}, \tag{12}$$

for all $(i,j) \in \mathbb{N}^+_R \times \mathbb{N}^+_R$ and for all $(k,l) \in \mathbb{N}^+_{N_T} \times \mathbb{N}^+_{N_T}$. In the above, $\eta^2 > 0$ and $\beta > 0$ are called the *shadowing power* and the *correlation distance*, respectively [32]. In this fashion, we will call $\gamma > 0$ and $\delta > 0$ the *correlation time* and the *BS*

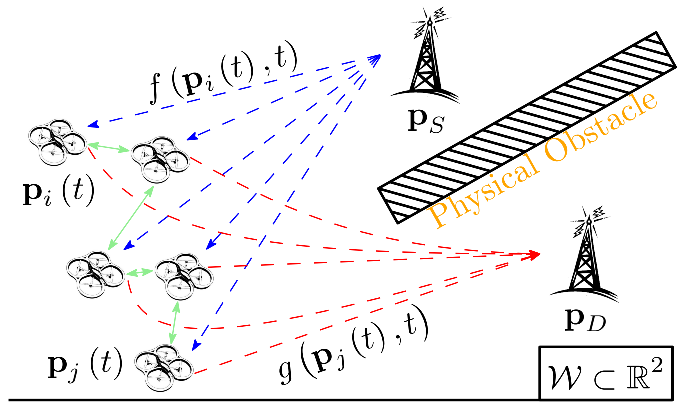

Figure 2: A case where source-relay and relay-destination links are likely to be correlated.

(Base Station) correlation, respectively. For later reference, let us define the (cross)covariance matrices in $\mathbb{S}^R$

$$\boldsymbol{\Sigma}_{SD}(k,l) \triangleq \mathbb{E}\left\{\boldsymbol{\sigma}_S(k)\,\boldsymbol{\sigma}^{\boldsymbol{T}}_D(l)\right\} + \mathbb{1}_{\{S\equiv D\}}\mathbb{1}_{\{k\equiv l\}}\sigma^2_\xi\mathbf{I}_R, \tag{13}$$

as well as

$$\boldsymbol{\Sigma}(k,l) \triangleq \begin{bmatrix}\boldsymbol{\Sigma}_{SS}(k,l) & \boldsymbol{\Sigma}_{SD}(k,l)\\ \boldsymbol{\Sigma}_{SD}(k,l) & \boldsymbol{\Sigma}_{DD}(k,l)\end{bmatrix} \in \mathbb{S}^{2R}, \tag{14}$$

for all $(k,l) \in \mathbb{N}^+_{N_T} \times \mathbb{N}^+_{N_T}$. Then, the covariance matrix of the joint distribution describing (10) can be expressed as

$$\boldsymbol{\Sigma} \triangleq \begin{bmatrix}\boldsymbol{\Sigma}(1,1) & \ldots & \boldsymbol{\Sigma}(1,N_T)\\ \vdots & \ddots & \vdots\\ \boldsymbol{\Sigma}(N_T,1) & \ldots & \boldsymbol{\Sigma}(N_T,N_T)\end{bmatrix} \in \mathbb{S}^{2RN_T}. \tag{15}$$

Of course, in order for $\boldsymbol{\Sigma}$ to be a valid covariance matrix, it must be at least positive semidefinite. If fact, for nearly all cases of interest, $\boldsymbol{\Sigma}$ is guaranteed to be strictly positive definite, as the following basic result suggests.

Lemma 2. (Positive (Semi)Definiteness of $\boldsymbol{\Sigma}$) *For all possible deterministic trajectories of the relays on $\mathcal{S}^R \times \mathbb{N}^+_{N_T}$, it is true that $\boldsymbol{\Sigma} \in \mathbb{S}^{2RN_T}_{++}$, as long as $\sigma^2_\xi \neq 0$. Otherwise, $\boldsymbol{\Sigma} \in \mathbb{S}^{2RN_T}_{+}$. In other words, as long as multipath (small-scale) fading is present in the channel response, the joint Gaussian distribution of* (10) *is guaranteed to be nonsingular.*

Proof of Lemma 2: See Appendix. ■

B. Model Justification & Extensions

As already mentioned, spatial dependence among the source-relay and relay-destination channel magnitudes (due to shadowing) is described via Gudmundson's model [32] (position related component in (11)), which has been very popular in the literature and also experimentally verified [23], [32], [33]. Second, the Laplacian type of temporal dependence among the same groups of channel magnitudes also constitutes a reasonable choice, in the sense that channel magnitudes are expected to be significantly correlated only for small time lags, whereas, for larger time lags, such dependence should decay at a fast rate. Of course, one could use any other positive (semi)definite kernel, without changing the statement and proof of Lemma 2. Third, the incorporation of the spherical/isotropic BS correlation term in our proposed general model (in (12)) can be justified by the existence of important cases where the source and destination might be close to each other and yet no direct link may exist between them. See, for instance, Fig. 2, where a "large" physical obstacle makes direct communication between the source and the destination impossible. Then, relay beamforming can be exploited in order to improve or maintain communication between the source and the destination, making intelligent use of available resources. In such cases, however, it is very likely that the shadowing parts of the source-relay and relay-destination links will be spatially and/or temporally correlated among each other, since shadowing is very much affected by the spatial characteristics of the terrain, which, in such cases, is common for both beamforming phases. Of course, by taking $\delta \to 0$, one recovers the generic/trivial case where the source-relay and relay-destination links are *mutually independent*.

IV. Problem Formulation

In this section, we formulate the spatially controlled relay beamforming problem, advocated in this paper.

A. Joint Scheduling of Communications & Controls

At each time slot $t \in \mathbb{N}^+_{N_T}$ and assuming the same carrier for all communication tasks, we employ a basic joint communication/decision making TDMA-like protocol, as follows:

1) The source broadcasts a pilot signal to the relays, which then estimate their channels relative to the source.
2) The same procedure is carried out for the channels relative to the destination.
3) Then, based on the estimated CSI, the relays beamform in AF mode (assume perfect CSI estimation).
4) Based on CSI received *so far*, strategic decision making is implemented, relay motion controllers are determined and relays are steered to their updated positions.

These actions are repeated for all N_T time slots. In order to simplify the presentation, we will additionally assume that, following Action 2, *the relays can forward all estimated CSI to the destination*, via a dedicated low rate channel. This simplifies information decoding since, in principle, channel effects can be mitigated at the destination. In such case, the effect of SINR maximization on the achievable Bit-Error-Rate (BER) follows well-known results, see, e.g., [34], [35].

Concerning relay kinematics, it is assumed that the relays obey the differential equation

$$\dot{\mathbf{p}}(\tau) \equiv \mathbf{u}(\tau), \quad \forall \tau \in [0, T], \tag{16}$$

where $\mathbf{u} \triangleq [\mathbf{u}_1 \ldots \mathbf{u}_R]^{\boldsymbol{T}} \in \mathcal{S}^R$, with $\mathbf{u}_i : [0, T] \to \mathcal{S}$ being the motion controller of relay $i \in \mathbb{N}^+_R$. Apparently, relay motion is in continuous time. However, assuming the relays may move *only after their controls have been determined and up to the start of the next time slot*, we can write

$$\mathbf{p}(t) \equiv \mathbf{p}(t-1) + \int_{\Delta\tau_{t-1}} \mathbf{u}_{t-1}(\tau)\, \mathrm{d}\tau, \quad \forall t \in \mathbb{N}^2_{N_T}, \tag{17}$$

with $\mathbf{p}(1) \equiv \mathbf{p}_{init}$, and where $\Delta\tau_t \subset \mathbb{R}$ and $\mathbf{u}_t : \Delta\tau_t \to \mathcal{S}^R$ denote the time interval that the relays are allowed to move in, and the respective relay controller, in each time slot $t \in \mathbb{N}^+_{N_T-1}$. It holds that $\mathbf{u}(\tau) \equiv \sum_{t \in \mathbb{N}^+_{N_T-1}} \mathbf{u}_t(\tau)\, \mathbb{1}_{\Delta\tau_t}(\tau)$, *where τ belongs in the first* $N_T - 1$ time slots. Of course, at each time slot t, the length of $\Delta\tau_t$, $|\Delta\tau_t|$, must be sufficiently small such that the temporal correlations of the CSI at adjacent time slots are sufficiently strong. These are controlled by the correlation time γ, which can be a function of the slot width. Therefore, relay velocity must be of the order of $(|\Delta\tau_t|)^{-1}$. In this work, though, for simplicity, we assume that the relays are not explicitly resource constrained, in terms of their motion.

Now, regarding the form of the relay motion controllers $\mathbf{u}_{t-1}(\tau), \tau \in \Delta\tau_{t-1}$, *given a goal position vector at time slot* t, $\mathbf{p}^o(t)$, it suffices to fix a path in $\mathcal{S}^R$, such that the points $\mathbf{p}^o(t)$ and $\mathbf{p}(t-1)$ are connected in at most time $|\Delta\tau_{t-1}|$. A generic choice for such a path is the straight line connecting $\mathbf{p}^o_i(t)$ and $\mathbf{p}_i(t-1)$, for all $i \in \mathbb{N}^+_R$. Thus, we may choose the relay controllers at time slot $t-1 \in \mathbb{N}^+_{N_T-1}$ as

$$\mathbf{u}^o_{t-1}(\tau) \triangleq \frac{1}{|\Delta\tau_{t-1}|}\left(\mathbf{p}^o(t) - \mathbf{p}(t-1)\right), \ \forall \tau \in \Delta\tau_{t-1}. \tag{18}$$

As a result, any motion control problem considered hereafter can be formulated in terms of specifying the goal relay positions at the next time slot, given their positions at the current time slot (and the observed CSI). For simplicity, we assume that collisions never occur, or that, if they do, there exists some transparent path planning / collision avoidance mechanism implemented at each relay, out of our direct control. Note that the *pure* path planning problem of *physically* moving the relays at each time slot is out of the scope of this paper.

Hereafter, let $\mathscr{C}(\mathcal{T}_t)$ denote the set of channel gains observed by the relays, *along the paths of their point trajectories* $\mathcal{T}_t \triangleq \{\mathbf{p}(1) \ldots \mathbf{p}(t)\}$, $t \in \mathbb{N}^+_{N_T}$. Then, $\mathcal{T}_t$ may be recursively updated as $\mathcal{T}_t \equiv \mathcal{T}_{t-1} \cup \{\mathbf{p}(t)\}$, for all $t \in \mathbb{N}^+_{N_T}$, with $\mathcal{T}_0 \triangleq \varnothing$. In a technically precise sense, $\{\mathscr{C}(\mathcal{T}_t)\}_{t \in \mathbb{N}^+_{N_T}}$ will also denote the filtration generated by the CSI observed at the relays, *along* $\mathcal{T}_t$, interchangeably. In other words, in case the trajectories of the relays are themselves random, then $\mathscr{C}(\mathcal{T}_t)$ denotes the σ-algebra generated by *both* the CSI observed up to and including time slot t *and* $\mathbf{p}(1) \ldots \mathbf{p}(t)$, for all $t \in \mathbb{N}^+_{N_T}$. Additionally, we define $\mathscr{C}(\mathcal{T}_0) \equiv \mathscr{C}(\{\varnothing\})$ as $\mathscr{C}(\mathcal{T}_0) \triangleq \{\varnothing, \Omega\}$, that is, as the trivial σ-algebra, and we may refer to time $t \equiv 0$, as a *dummy time slot*, by convention.

Remark 3. Note that, in our work, we assume that communication and motion control do not happen simultaneously. This means that, in a practical setting, the source, destination and all relays, *while communicating*, are either completely still, or they move sufficiently slowly, such that the local spatial and temporal changes of the wireless channel are negligible, as are Doppler shift effects. Consequently, temporal small *and* large scale channel variations are only due to changes in the physical characteristics of the space, which happen at considerably slower frequency than the rate of actual communication. Apparently, there is a natural interplay between relay velocity and the relative rate of change of the communication channel. The challenge is to identify a fair tradeoff between a reasonable relay velocity, and a communication window of appropriate size, which would enable faithful channel prediction. The width of the communication window depends significantly on the spatial characteristics of the terrain in the specific application, which also determine the sampling rate employed for channel model training. Further, the interval occupied by relay motion within each time slot should be as small as possible. In theory, for a given relay velocity, the relays could move to any position up to which the channel remains correlated. However, as the *per time slot* rate of communications depends on the relay velocity (characterizing system throughput), the relays should move to much smaller distances within the slot. ■

B. Spatially Controlled SINR Maximization at the Destination

Next, we propose a 2-stage stochastic programming approach, optimizing network QoS by optimally selecting beamforming weights *and* relay positions, on a *per time slot* basis.

Optimization of Beamforming Weights: At time slot $t \in \mathbb{N}^+_{N_T}$, *given* CSI in $\mathscr{C}(\mathcal{T}_t)$, we formulate the problem [3], [6]

$$\begin{array}{ll} \underset{\boldsymbol{w}(t) \triangleq [w_1^*(t) \ldots w_R^*(t)]^{\boldsymbol{T}}}{\text{maximize}} & \dfrac{\mathbb{E}\{P_S(t)|\mathscr{C}(\mathcal{T}_t)\}}{\mathbb{E}\{P_{I+N}(t)|\mathscr{C}(\mathcal{T}_t)\}} \\ \text{subject to} & \mathbb{E}\{P_R(t)|\mathscr{C}(\mathcal{T}_t)\} \le P_c \end{array}, \tag{19}$$

where $P_R(t)$, $P_S(t)$ and $P_{I+N}(t)$ denote the random instantaneous power at the relays, that of the signal component and that of the interference plus noise component at the destination (see (1)), respectively and where $P_c > 0$ denotes the total available relay transmission power. Exploiting mutual independence regarding CSI related to the source and destination, respectively, (19) can be reexpressed analytically as [3]

$$\begin{array}{ll} \underset{\boldsymbol{w}(t)}{\text{maximize}} & \dfrac{\boldsymbol{w}^{\boldsymbol{H}}(t)\,\mathbf{R}(\mathbf{p}(t),t)\,\boldsymbol{w}(t)}{\sigma_D^2 + \boldsymbol{w}^{\boldsymbol{H}}(t)\,\mathbf{Q}(\mathbf{p}(t),t)\,\boldsymbol{w}(t)} \\ \text{subject to} & \boldsymbol{w}^{\boldsymbol{H}}(t)\,\mathbf{D}(\mathbf{p}(t),t)\,\boldsymbol{w}(t) \le P_c \end{array}, \tag{20}$$

where, dropping the dependence on $(\mathbf{p}(t),t)$ or t for brevity,

$$\mathbf{D} \triangleq P_0 \text{diag}\left(\left[|f_1|^2\ |f_2|^2\ \ldots\ |f_R|^2\right]^{\boldsymbol{T}}\right) + \sigma^2 \mathbf{I}_R \in \mathbb{S}^R_{++}, \tag{21}$$

$$\mathbf{R} \triangleq P_0 \mathbf{h}\mathbf{h}^{\boldsymbol{H}} \in \mathbb{S}^R_+, \text{ with } \mathbf{h} \triangleq [f_1 g_1\ f_2 g_2\ \ldots\ f_R g_R]^{\boldsymbol{T}} \text{ and} \tag{22}$$

$$\mathbf{Q} \triangleq \sigma^2 \text{diag}\left(\left[|g_1|^2\ |g_2|^2\ \ldots\ |g_R|^2\right]^{\boldsymbol{T}}\right) \in \mathbb{S}^R_{++}. \tag{23}$$

Note that the program (20) is *always feasible, as long as P_c is nonnegative*. It is well known that the optimal value of (20) can be expressed in closed form as [3]

$$\begin{aligned} V_t &\equiv V(\mathbf{p}(t),t) \qquad (24) \\ &\triangleq P_c \lambda_{max}\left(\left(\sigma_D^2 \mathbf{I}_R + P_c \mathbf{D}^{-1/2}\mathbf{Q}\mathbf{D}^{-1/2}\right)^{-1} \mathbf{D}^{-1/2}\mathbf{R}\mathbf{D}^{-1/2}\right), \end{aligned}$$

for all $t \in \mathbb{N}^+_{N_T}$. Exploiting the structure of the matrices involved, V_t may also be expressed *analytically* as [6]

$$\begin{aligned} V_t &\equiv \sum_{i \in \mathbb{N}^+_R} \frac{P_c P_0 |f(\mathbf{p}_i(t),t)|^2 |g(\mathbf{p}_i(t),t)|^2}{P_0 \sigma_D^2 |f(\mathbf{p}_i(t),t)|^2 + P_c \sigma^2 |g(\mathbf{p}_i(t),t)|^2 + \sigma^2 \sigma_D^2} \\ &\triangleq \sum_{i \in \mathbb{N}^+_R} V_I(\mathbf{p}_i(t),t), \quad \forall t \in \mathbb{N}^+_{N_T}. \end{aligned} \tag{25}$$

This analytical representation of the optimal value V_t will be of crucial importance in our subsequent development.

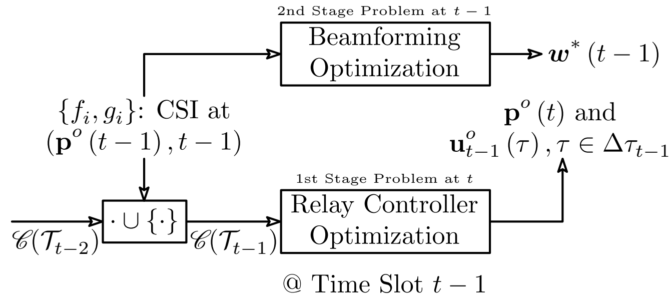

Figure 3: 2-Stage optimization of beamforming weights and relay motion controls. The variable $\boldsymbol{w}^*(t-1)$ denotes the optimal beamforming weights, selected at time slot $t-1$.

Optimization of Beamformer Positions: At time slot $t-1$, we are interested in choosing relay positions at time slot t, such that V_t is maximized. However, at time slot $t-1$, we are only given $\mathscr{C}(\mathcal{T}_{t-1})$, which does not encode future CSI, revealed at time slot t. Therefore, *exact* optimization of the relay positions at the next time slot is *impossible*. Nevertheless, it would be reasonable to search for the best decision on the positions of the relays at time slot t, such that V_t is maximized *in expectation, relative to $\mathscr{C}(\mathcal{T}_{t-1})$*.

But what *relative to $\mathscr{C}(\mathcal{T}_{t-1})$* means quantitatively? Since, at time slot $t-1$, deterministic optimization of V_t with respect to $\mathbf{p}(t)$ is impossible, it makes sense to consider optimizing *a projection of V_t onto the space of all measurable functions of $\mathscr{C}(\mathcal{T}_{t-1})$*. Since, for every $\mathbf{p}(t) \in \mathcal{S}^R$, V_t is of finite variance, it is then reasonable to consider orthogonal projections or, in other words, the Minimum Mean Square Error (MMSE) predictor of V_t given $\mathscr{C}(\mathcal{T}_{t-1})$. One then optimizes the *random utility* $\mathbb{E}\{V_t|\mathscr{C}(\mathcal{T}_{t-1})\}$ relative to *the point* $\mathbf{p}(t)$, resulting in the 2-*stage stochastic program* [36]

$$\begin{array}{ll} \underset{\mathbf{p}(t)}{\text{maximize}} & \mathbb{E}\left\{V_t \equiv \displaystyle\sum_{i \in \mathbb{N}^+_R} V_I(\mathbf{p}_i(t),t) \,\middle|\, \mathscr{C}(\mathcal{T}_{t-1})\right\} \\ \text{subject to} & \mathbf{p}(t) \in \mathcal{C}(\mathbf{p}^o(t-1)) \end{array}, \tag{26}$$

solved at time slot $t-1 \in \mathbb{N}^+_{N_T-1}$, where $\mathbf{p}^o(1) \in \mathcal{S}^R$ is a known constant, representing the initial positions of the relays and $\mathcal{C}(\mathbf{p}^o(t-1)) \subseteq \mathcal{S}^R$ denotes a finite set representing a spatially feasible neighborhood around the point $\mathbf{p}^o(t-1) \in \mathcal{S}^R$, the (possibly optimal) decision vector *selected* at time $t-2 \in \mathbb{N}_{N_T-2}$ (recall that $t \equiv 0$ denotes a dummy time slot). For instance, $\mathcal{C}$ might prevent relays from colliding with each other, at their goal positions. If, further, $\mathcal{C}$ is allowed itself to depend on t (for simplicity it is not), it could also prevent collisions of relays with other obstacles in the space (at the goal positions of the relays). In the relevant literature, the map $\mathcal{C}(\cdot)$ is referred to as a *finite-valued multifunction*, and we write $\mathcal{C} : \mathcal{S}^R \rightrightarrows \mathcal{S}^R$ [36]. Additionally, problems (26) and (20) are referred to as the *first-stage problem* and the *second-stage problem*, respectively [36]. A block diagram of the proposed approach is shown in Fig. 3.

As compared with traditional, stationary AF beamforming, the additional challenge in our spatially controlled system described above is that, *while using the same CSI as in the stationary case*, each relay *(MMSE-optimally) predicts* *optimal beamforming performance* in its vicinity, and moves to an *optimally selected location*, presuming the validity of the proposed spatiotemporal channel model. As explained in Remark 3, this requires a sufficiently slowly varying channel as compared to relay motion, and/or motion constrained within small steps. As a prototypical example, one can think of drones optimally moving in small steps in order to improve communication quality (also see Fig. 1).

C. Motion Policies & The Interchangeability Principle

Before proceeding with the development of techniques for solving (26), we discuss an important *variational* property of (26), related to the *long-term performance* of the proposed spatially controlled beamforming system. Our discussion is based on the exploitation of the so-called *Interchangeability Principle (IP)* [36]–[40], also known as the *Fundamental Lemma of Stochastic Control (FLSC)* [41], [42]. The IP is not a single mathematical statement, but refers to a *family* of technical results, which provide conditions permitting interchange of expectation and max/minimization in general stochastic programs.

A version of the IP, which fits the structural framework under which the pointwise (over constants) first-stage problem (26) is formulated in this paper, is rigorously established in [40] (for details, the reader is referred to the analysis of [40], but this is out of the scope of this paper). Specifically, the IP implies that (26) is *exchangeable* by the *variational problem*

$$\begin{array}{ll} \underset{\mathbf{p}(t)}{\mathrm{maximize}} & \mathbb{E}\left\{V_t\right\} \\ \mathrm{subject\ to} & \mathbf{p}\left(t\right) \in \mathcal{C}\left(\mathbf{p}^o\left(t-1\right)\right) \\ & \mathbf{p}\left(t\right)\ \text{ is } \mathscr{C}\left(\mathcal{T}_{t-1}\right)\text{-measurable} \end{array}, \qquad (27)$$

to be solved at each $t-1 \in \mathbb{N}^+_{N_T-1}$. The crucial difference between (27) and our original problem (26) is that, in the former, optimization of the *unconditional expectation* of V_t is considered, over *all (measurable) mappings* of the variables generating $\mathscr{C}\left(\mathcal{T}_{t-1}\right)$ to $\mathcal{C}\left(\mathbf{p}^o\left(t-1\right)\right)$. This implies that, in (27), $\mathbf{p}\left(t\right)$ is a *function* of all CSI and motion controls up to and including time slot $t-1$, whereas, in (26), $\mathbf{p}\left(t\right)$ is a *point*, since all variables generating $\mathscr{C}\left(\mathcal{T}_{t-1}\right)$ are *fixed before decision making*. Aligned with the literature, any feasible decision $\mathbf{p}\left(t\right)$ in (27) will be called an *(admissible) policy*, or a *decision rule*.

Exchangeability of (26) and (27) is understood in the sense that the optimal value of (27), which is a number, coincides with the *expectation* of the optimal value of (26), which is a measurable function of $\mathscr{C}\left(\mathcal{T}_{t-1}\right)$ (and fixed for every realization of the variables generating $\mathscr{C}\left(\mathcal{T}_{t-1}\right)$). In other words, maximization is *interchangeable* with integration, in the sense that

$$\sup_{\mathbf{p}(t)\in\mathcal{D}_t} \mathbb{E}\{V_t\} \equiv \mathbb{E}\left\{ \sup_{\mathbf{p}(t)\in\mathcal{C}\left(\mathbf{p}^o(t-1)\right)} \mathbb{E}\{V_t\,|\mathscr{C}\left(\mathcal{T}_{t-1}\right)\}\right\}, \quad (28)$$

for all $t \in \mathbb{N}^2_{N_T}$, where $\mathcal{D}_t$ denotes the set of feasible decisions for (27). What is more, owing to our assumption that the control space $\mathcal{S}$ is finite, the IP guarantees that *every* optimal solution to the original stochastic program (26) is feasible and thus, optimal, for (27).

Remark 4. (***Why Policies?***) Although problem (26) is intuitively justified, dependence of its objective (26) on $\mathscr{C}\left(\mathcal{T}_{t-1}\right)$ does not *always* render it a useful optimality criterion. This is because the objective of (26) quantifies the performance of a *single* decision, *only conditioned* on $\mathscr{C}\left(\mathcal{T}_{t-1}\right)$. In other words, the objective of (26) *does not* quantify the performance of *a policy*. To do that, any reasonable performance criterion should assign *a number* to each policy, ranking its quality for all possible values of the variables generating $\mathscr{C}\left(\mathcal{T}_{t-1}\right)$. The expected utility $\mathbb{E}\left\{V_t\right\}$ of problem (27) constitutes a suitable such criterion. And by the IP, (27) may be reduced to (26), which can thus be regarded as a proxy for solving the former.

In our spatially controlled beamforming problem, the implications of the IP may be intuitively demonstrated as follows. By solving our original problem (26), each relay chooses their optimal motion control, *given* the information collected *so far* by the network. Although this is indeed a reasonable thing to do at each time slot *from an opportunistic perspective*, it does not immediately imply that it is also a good thing to do for *(almost) every possible realization* of the information that could be potentially collected by the network. The IP confirms this assertion by showing that, in fact, optimizing *instantaneous* network QoS by solving (26), also optimizes network QoS *on average*, with respect to all admissible relay motion policies, at each particular time slot (problem (27)).

The main reason justifying our interest in policies is that, *except for* instantaneous performance, one should also be interested in *long-term performance* of the beamforming system. In other words, it should be possible to assess system performance if the system is used repeatedly over time, e.g., periodically (every hour, day) or on demand. For example, consider a beamforming system, which operates for N_T time slots and independently *restarts* its operation at time slots kN_T+1, for k in some subset of $\mathbb{N}^+$. This might be practically essential for maintaining system stability over time, saving on resources, etc. It is then clear that merely quantifying the performance of individual decisions is incomplete, from an operational point of view; simply, the random utility approach (that is, (26)) quantifies performance *only along a path of the observed information*, $\mathscr{C}\left(\mathcal{T}_{t-1}\right)$, for $t \in \mathbb{N}^+_{N_T}$. This issue is more profound when channel observations taking specific values correspond to events of zero measure (as with the channel model of Section III). On the contrary, it is of interest to quantify system performance when decisions are made for all outcomes of the sample space Ω. This necessitates assessment of different policies (decision rules), and this is only possible by considering variational optimization (policy search) problems, such as (27).

Further, consideration of the variational program (27) is practically motivated, as well. Simulating repeatedly the system and by the Law of Large Numbers, one may obtain excellent estimates of the expected performance of the system, quantified by the chosen utility. Therefore, the systematic experimental assessment of a particular sequence of policies (one for each time slot) is readily possible (see Section VII). Apparently, this is impossible to perform by adopting the random utility approach, since, in this case, system performance is quantified via a real valued (in general) random quantity.

$$\boldsymbol{m}_{1:t-1} \triangleq \left[\boldsymbol{F}^{\boldsymbol{T}}(1)\ \boldsymbol{G}^{\boldsymbol{T}}(1)\ \ldots\ \boldsymbol{F}^{\boldsymbol{T}}(t-1)\ \boldsymbol{G}^{\boldsymbol{T}}(t-1)\right]^{\boldsymbol{T}} \in \mathbb{R}^{2R(t-1)\times 1} \tag{29}$$

$$\boldsymbol{\mu}_{1:t-1} \triangleq \left[\boldsymbol{\alpha}_S(\mathbf{p}(1))\ \boldsymbol{\alpha}_D(\mathbf{p}(1))\ \ldots\ \boldsymbol{\alpha}_S(\mathbf{p}(t-1))\ \boldsymbol{\alpha}_D(\mathbf{p}(t-1))\right]^{\boldsymbol{T}} \ell \in \mathbb{R}^{2R(t-1)\times 1} \tag{30}$$

$$\boldsymbol{c}_{1:t-1}^{F(G)}(\mathbf{p}) \triangleq \left[\boldsymbol{c}_1^{F(G)}(\mathbf{p})\ \ldots\ \boldsymbol{c}_{t-1}^{F(G)}(\mathbf{p})\right] \in \mathbb{R}^{1\times 2R(t-1)} \tag{31}$$

$$\boldsymbol{c}_k^{F(G)}(\mathbf{p}) \triangleq \left[\left\{\mathbb{E}\left\{\sigma_{S(D)}(\mathbf{p},t)\,\sigma_S^j(k)\right\}\right\}_{j\in\mathbb{N}_R^+}\ \left\{\mathbb{E}\left\{\sigma_{S(D)}(\mathbf{p},t)\,\sigma_D^j(k)\right\}\right\}_{j\in\mathbb{N}_R^+}\right] \in \mathbb{R}^{1\times 2R},\ \forall k\in\mathbb{N}_{t-1}^+ \tag{32}$$

$$\boldsymbol{\Sigma}_{1:t-1} \triangleq \begin{bmatrix} \boldsymbol{\Sigma}(1,1) & \cdots & \boldsymbol{\Sigma}(1,t-1) \\ \vdots & \ddots & \vdots \\ \boldsymbol{\Sigma}(t-1,1) & \cdots & \boldsymbol{\Sigma}(t-1,t-1) \end{bmatrix} \in \mathbb{S}_{++}^{2R(t-1)} \tag{33}$$

V. Near-Optimal Beamformer Motion Control

Problem (26) enjoys rather favorable structure. In particular, we readily observe that (26) is separable. In fact, given that, *for each $t\in\mathbb{N}_{N_T-1}^+$, decisions taken and CSI collected so far are available to all relays*, (26) can be solved in a *completely distributed fashion at the relays*, with the i-th relay being responsible for solving the program

$$\begin{array}{ll} \underset{\mathbf{p}}{\text{maximize}} & \mathbb{E}\left\{V_I(\mathbf{p},t)|\mathscr{C}(\mathcal{T}_{t-1})\right\} \\ \text{subject to} & \mathbf{p}\in\mathcal{C}_i(\mathbf{p}^o(t-1)) \end{array}, \tag{34}$$

at each $t-1\in\mathbb{N}_{N_T-1}^+$, where $\mathcal{C}_i:\mathbb{R}^2\rightrightarrows\mathbb{R}^2$ denotes the corresponding section of $\mathcal{C}$, for each $i\in\mathbb{N}_R^+$. Note that no local exchange of intermediate results is required among relays; given the available information, each relay independently solves its own subproblem. It is also evident that apart from the obvious difference in the feasible set, the optimization problems at each of the relays are identical.

However, problem (34) presents an important technical difficulty: Its objective involves the evaluation of a conditional expectation of a well defined ratio of almost surely positive random variables, which is *impossible to perform analytically*. Hence, it is necessary to resort to well behaved and computationally efficient *surrogates* to problem (34). Next, we present *three near-optimal* such approaches. The first two rely on *global* function approximation techniques, and achieve excellent empirical performance. The third approach is based on *local* approximations of the objective of (34) and, although strictly suboptimal, it is extremely computationally efficient.

All proposed approximations to the stochastic program (34) will be based on the following technical, though simple, result.

Lemma 5. (Big Expectations) *Under the assumptions of the wireless channel model introduced in Section III, it is true that, at any $\mathbf{p}\in\mathcal{S}$,*

$$\left.\begin{bmatrix} F(\mathbf{p},t) \\ G(\mathbf{p},t)\end{bmatrix}\right|\mathscr{C}(\mathcal{T}_{t-1}) \sim \mathcal{N}\left(\boldsymbol{\mu}_{t|t-1}^{F,G}(\mathbf{p}),\boldsymbol{\Sigma}_{t|t-1}^{F,G}(\mathbf{p})\right), \tag{35}$$

for all $t\in\mathbb{N}_{N_T}^2$, and where we define

$$\boldsymbol{\mu}_{t|t-1}^{F,G}(\mathbf{p}) \triangleq \left[\mu_{t|t-1}^F(\mathbf{p})\ \mu_{t|t-1}^G(\mathbf{p})\right]^{\boldsymbol{T}}, \tag{36}$$

$$\begin{aligned} \mu_{t|t-1}^F(\mathbf{p}) &\triangleq \alpha_S(\mathbf{p})\,\ell \\ &\quad + \boldsymbol{c}_{1:t-1}^F(\mathbf{p})\,\boldsymbol{\Sigma}_{1:t-1}^{-1}\big(\boldsymbol{m}_{1:t-1}-\boldsymbol{\mu}_{1:t-1}\big)\in\mathbb{R}, \end{aligned} \tag{37}$$

$$\begin{aligned} \mu_{t|t-1}^G(\mathbf{p}) &\triangleq \alpha_D(\mathbf{p})\,\ell \\ &\quad + \boldsymbol{c}_{1:t-1}^G(\mathbf{p})\,\boldsymbol{\Sigma}_{1:t-1}^{-1}\big(\boldsymbol{m}_{1:t-1}-\boldsymbol{\mu}_{1:t-1}\big)\in\mathbb{R}\ \textit{ and} \end{aligned} \tag{38}$$

$$\begin{aligned} \boldsymbol{\Sigma}_{t|t-1}^{F,G}(\mathbf{p}) &\triangleq \begin{bmatrix} \eta^2+\sigma_\xi^2 & \eta^2 e^{-\frac{\|\mathbf{p}_S-\mathbf{p}_D\|_2}{\delta}} \\ \eta^2 e^{-\frac{\|\mathbf{p}_S-\mathbf{p}_D\|_2}{\delta}} & \eta^2+\sigma_\xi^2 \end{bmatrix} \\ &\quad - \begin{bmatrix} \boldsymbol{c}_{1:t-1}^F(\mathbf{p}) \\ \boldsymbol{c}_{1:t-1}^G(\mathbf{p})\end{bmatrix} \boldsymbol{\Sigma}_{1:t-1}^{-1} \begin{bmatrix} \boldsymbol{c}_{1:t-1}^F(\mathbf{p}) \\ \boldsymbol{c}_{1:t-1}^G(\mathbf{p})\end{bmatrix}^{\boldsymbol{T}} \in\mathbb{S}_{++}^2, \end{aligned} \tag{39}$$

with $\boldsymbol{m}_{1:t-1}$, $\boldsymbol{\mu}_{1:t-1}$, $\boldsymbol{c}_{1:t-1}^F(\mathbf{p})$, $\boldsymbol{c}_{1:t-1}^G(\mathbf{p})$, $\boldsymbol{c}_k^F(\mathbf{p})$, $\boldsymbol{c}_k^G(\mathbf{p})$ and $\boldsymbol{\Sigma}_{1:t-1}$ defined as in (29), (30), (31), (32), and (33) respectively (top of page), for all $(\mathbf{p},t)\in\mathcal{S}\times\mathbb{N}_{N_T}^2$. Further, for every choice of $(m,n)\in\mathbb{Z}\times\mathbb{Z}$, the conditional correlation of the fields $|f(\mathbf{p},t)|^m$ and $|g(\mathbf{p},t)|^n$ relative to $\mathscr{C}(\mathcal{T}_{t-1})$ may be expressed in closed form as

$$\begin{aligned} &\mathbb{E}\left\{|f(\mathbf{p},t)|^m\,|g(\mathbf{p},t)|^n|\,\mathscr{C}(\mathcal{T}_{t-1})\right\} \\ &\equiv 10^{(m+n)\rho/20}\exp\Bigg(\frac{\log(10)}{20}\begin{bmatrix} m\\ n\end{bmatrix}^{\boldsymbol{T}}\boldsymbol{\mu}_{t|t-1}^{F,G}(\mathbf{p}) \\ &\qquad + \left(\frac{\log(10)}{20}\right)^2\begin{bmatrix} m\\ n\end{bmatrix}^{\boldsymbol{T}}\boldsymbol{\Sigma}_{t|t-1}^{F,G}(\mathbf{p})\begin{bmatrix} m\\ n\end{bmatrix}\Bigg), \end{aligned} \tag{40}$$

at any $\mathbf{p}\in\mathcal{S}$ and for all $t\in\mathbb{N}_{N_T}^2$.

Proof of Lemma 5: See Appendix. ■

The detailed description of the proposed techniques for efficiently approximating our base problem (34) now follows.

Sample Average Approximation (SAA): This is the direct Monte Carlo approach, where, *at worst*, existence of a *sampling, or pseudosampling mechanism at each relay* is assumed, capable of generating samples from a bivariate Gaussian measure. We may then observe that the objective of (34) can be represented, for all $t\in\mathbb{N}_{N_T}^2$, via a Lebesgue integral as

$$\begin{aligned} &\mathbb{E}\left\{V_I(\mathbf{p},t)|\,\mathscr{C}(\mathcal{T}_{t-1})\right\} \\ &\quad = \int_{\mathbb{R}^2} r(\boldsymbol{x})\,\mathcal{N}\left(\boldsymbol{x};\boldsymbol{\mu}_{t|t-1}^{F,G}(\mathbf{p}),\boldsymbol{\Sigma}_{t|t-1}^{F,G}(\mathbf{p})\right)\mathrm{d}\boldsymbol{x}, \end{aligned} \tag{41}$$

for any choice of $\mathbf{p}\in\mathcal{S}$, where $\mathcal{N}(\cdot;\boldsymbol{\mu},\boldsymbol{\Sigma}):\mathbb{R}^2\to\mathbb{R}_{++}$ denotes the bivariate Gaussian density, with mean $\boldsymbol{\mu}\in\mathbb{R}^{2\times 1}$ and covariance $\boldsymbol{\Sigma}\in\mathbb{S}_+^{2\times 2}$, and the function $r:\mathbb{R}^2\to\mathbb{R}_{++}$ is defined exploiting the bijective formula (8) as

$$r(\boldsymbol{x}) \triangleq \frac{P_c P_0 10^{\rho/10}\left[\exp(x_1+x_2)\right]^{\varsigma}}{P_0\sigma_D^2[\exp(x_1)]^{\varsigma}+P_c\sigma^2[\exp(x_2)]^{\varsigma}+10^{-\frac{\rho}{10}}\sigma^2\sigma_D^2}, \tag{42}$$

for all $\boldsymbol{x} \equiv (x_1, x_2) \in \mathbb{R}^2$, where $\varsigma \triangleq \log(10)/10$. By a simple change of variables, it is also true that

$$\mathbb{E}\left\{V_I(\mathbf{p},t)|\mathscr{C}(\mathcal{T}_{t-1})\right\} = \int_{\mathbb{R}^2} r\left(\sqrt{\boldsymbol{\Sigma}^{F,G}_{t|t-1}(\mathbf{p})}\boldsymbol{x} + \boldsymbol{\mu}^{F,G}_{t|t-1}(\mathbf{p})\right)\mathcal{N}(\boldsymbol{x};\mathbf{0},\mathbf{I}_2)\,\mathrm{d}\boldsymbol{x}, \quad (43)$$

for all $\mathbf{p} \in \mathcal{S}$ and $t \in \mathbb{N}^2_{N_T}$.

Now, for each relay $i \in \mathbb{N}^+_R$, at each $t \in \mathbb{N}^+_{N_T-1}$ and for some $S \in \mathbb{N}^+$, let $\left\{\boldsymbol{x}^j_{i,t}\right\}_{j\in\mathbb{N}^+_S}$ be a sequence of independent random elements in $\mathbb{R}^2$, such that $\boldsymbol{x}^j_{i,t} \sim \mathcal{N}(\mathbf{0},\mathbf{I}_2)$, for all $j \in \mathbb{N}^+_S$. We also assume that all such sequences are mutually independent of the channel fields F and G. Then, by defining the sample average estimate

$$\mathsf{S}_S(\mathbf{p},t) \triangleq \frac{1}{S}\sum_{j\in\mathbb{N}^+_S} r\left(\sqrt{\boldsymbol{\Sigma}^{F,G}_{t|t-1}(\mathbf{p})}\boldsymbol{x}^i_{j,t-1} + \boldsymbol{\mu}^{F,G}_{t|t-1}(\mathbf{p})\right), \quad (44)$$

the SAA of our initial problem (34) is formulated as

$$\boxed{\begin{array}{ll} \underset{\mathbf{p}}{\text{maximize}} & \mathsf{S}_S(\mathbf{p},t) \\ \text{subject to} & \mathbf{p} \in \mathcal{C}_i(\mathbf{p}^o(t-1)) \end{array}}\,, \quad (45)$$

at relay $i \in \mathbb{N}^+_R$, solved at each $t-1 \in \mathbb{N}^+_{N_T-1}$. Although detailed analysis is out of the scope of this paper, we should mention that, because the feasible of set of the SAA (45) is not only compact but, even more, finite, and because its objective is (obviously) continuous relative to $\mathbf{p}$, the optimal solution of (45) possesses various strong asymptotic guarantees in terms of convergence to the optimal solution of the original problem, as $S \to \infty$. For details, the reader is referred to ([36], Chapter 5). On the downside, the SAA approach requires Monte Carlo sampling, which might be restrictive in certain scenarios.

Note that we have *not explicitly assumed* mutual independence among the sequences $\left\{\boldsymbol{x}^j_{i,t}\right\}_j$, for each i and each t. This means that one could generate *one sequence for all relays*, per time slot, or even further, *one sequence for all relays, for all times slots altogether*. Such sampling schemes are totally valid, since, on the one hand, all SAAs of the form (45) are solved independently at each relay, at each time slot while, on the other hand, Monte Carlo sampling is assumed to be independent from the spatiotemporal fields F and G. Such sampling schemes are very efficient for practical purposes, as they relax or even eliminate the need for (pseudo)random sampling at *every* individual relay. As we will see in the numerical simulations presented later in Section VII, this approach exhibits excellent empirical performance.

Gauss-Hermite Quadrature (GHQ): Similarly to the concept of the SAA, the GHQ constitutes a global approximation technique, with the distinctive difference that the latter is *purely deterministic*; by construction, no sampling is required. However, there is a price paid in terms of generality, since GHQ is specially designed for the approximate computation of multidimensional *Gaussian* integrals, involving a function of choice, times a term of the form $\exp(-x^2)$. Similarly to the SAA, it can be shown that the objective of (34) may be closely approximated by the double summation formula (see Section IV in [26])

$$\mathsf{Q}_Q(\mathbf{p},t) \triangleq \sum_{i\in\mathbb{N}^+_Q}\varpi_i\sum_{j\in\mathbb{N}^+_Q}\varpi_j r\left(\sqrt{\boldsymbol{\Sigma}^{F,G}_{t|t-1}(\mathbf{p})}\boldsymbol{q}_{i,j} + \boldsymbol{\mu}^{F,G}_{t|t-1}(\mathbf{p})\right), \quad (46)$$

where $Q \in \mathbb{N}^+$ is called the *quadrature resolution*, each plane vector $\boldsymbol{q}_{i,j} \triangleq \left[q_i\, q_j\right]^{\boldsymbol{T}} \in \mathbb{R}^{2\times 1}$ denotes the (i,j)-th *quadrature point* and the tuple $(\varpi_i, \varpi_j) \in \mathbb{R}^{2\times 1}$ contains the respective *weighting coefficients*, for all $(i,j) \in \mathbb{N}^+_Q \times \mathbb{N}^+_Q$. As already seen by their definitions, both sets of quadrature points and weighting coefficients are selected *independently in each dimension*. Quadrature points and weighting coefficients are deterministic and determined *apriori*, via the following procedure [26], [43]. Consider a matrix $\boldsymbol{J} \in \mathbb{R}^{Q\times Q}$, such that

$$\boldsymbol{J}(i,j) \triangleq \sqrt{\min\{i,j\}/2}\,\mathbb{1}_{\{|j-i|\equiv 1\}}, \quad (47)$$

for all $(i,j) \in \mathbb{N}^+_Q \times \mathbb{N}^+_Q$. That is, $\boldsymbol{J}$ constitutes a hollow, tridiagonal, symmetric matrix. Let $\lambda_i(\boldsymbol{J}) \in \mathbb{R}$ and $\boldsymbol{v}_i(\boldsymbol{J}) \in \mathbb{R}^{M\times 1}$ denote the i-th eigenvalue and respective *normalized* eigenvector of $\boldsymbol{J}$, for all $i \in \mathbb{N}^+_Q$. Then, simply, quadrature points and weighting coefficients are selected as

$$q_i \equiv \sqrt{2}\lambda_i(\boldsymbol{J}) \text{ and } \varpi_i \equiv (\boldsymbol{v}_i(\boldsymbol{J})(1))^2, \ \forall i \in \mathbb{N}^+_Q. \quad (48)$$

In (48), $\boldsymbol{v}_i(\boldsymbol{J})(1)$ denotes the first entry of $\boldsymbol{v}_i(\boldsymbol{J})$, $i \in \mathbb{N}^+_Q$.

Under the above considerations, the GHQ approximation of the original pointwise problem (34) is formulated as

$$\boxed{\begin{array}{ll} \underset{\mathbf{p}}{\text{maximize}} & \mathsf{Q}_Q(\mathbf{p},t) \\ \text{subject to} & \mathbf{p} \in \mathcal{C}_i(\mathbf{p}^o(t-1)) \end{array}}\,, \quad (49)$$

solved at relay $i \in \mathbb{N}^+_R$, at each time $t-1 \in \mathbb{N}^+_{N_T-1}$. Because quadrature points and weighting coefficients are carefully selected specifically for evaluating integrals of the form of (43), the GHQ approach yields *extremely accurate approximations* for *rather small values of the quadrature resolution* Q, thus successfully compensating for the presence of a double summation on the RHS of (46). This feature of the GHQ approach is clearly demonstrated in the numerical results of Section VII, which also explicitly verify that the empirical performance of the GHQ and the SAA is essentially identical.

Method of Statistical Differentials (MSD): Lastly, we present a less precise, however very computationally efficient technique for approximating $\mathbb{E}\left\{V_I(\mathbf{p},t)|\mathscr{C}(\mathcal{T}_{t-1})\right\}$. Now we exploit (40). To begin with, observe that V_I can expressed as

$$V_I(\mathbf{p},t) \equiv \frac{1}{V_{II}(\mathbf{p},t)} \quad (50)$$
$$\triangleq \frac{1}{\dfrac{\sigma_D^2}{P_c}|g(\mathbf{p},t)|^{-2} + \dfrac{\sigma^2}{P_0}|f(\mathbf{p},t)|^{-2} + \dfrac{\sigma^2\sigma_D^2}{P_cP_0}|f(\mathbf{p},t)|^{-2}|g(\mathbf{p},t)|^{-2}}$$

for all $(\mathbf{p},t) \in \mathcal{S}\times\mathbb{N}^+_{N_T}$. Motivated by the rational expression (50), our approach will be based on the so-called *Method of Statistical Differentials (MSD)* ([27], Section 3.14.2).

Specifically, for $t \in \mathbb{N}^2_{N_T}$, the MSD locally approximates the conditional expectation $\mathbb{E}\{V_I(\mathbf{p},t)|\mathscr{C}(\mathcal{T}_{t-1})\}$ around the function $\mathbb{E}\{V_{II}(\mathbf{p},t)|\mathscr{C}(\mathcal{T}_{t-1})\}$ via the *order*-1 surrogate

$$\mathsf{T}^E_1(\mathbf{p},t) \triangleq \frac{1}{\mathbb{E}\{V_{II}(\mathbf{p},t)|\mathscr{C}(\mathcal{T}_{t-1})\}}, \tag{51}$$

or via the *order*-2 surrogate

$$\mathsf{T}^E_2(\mathbf{p},t) \triangleq \frac{\mathbb{E}\left\{(V_{II}(\mathbf{p},t))^2\middle|\mathscr{C}(\mathcal{T}_{t-1})\right\}}{(\mathbb{E}\{V_{II}(\mathbf{p},t)|\mathscr{C}(\mathcal{T}_{t-1})\})^3}, \tag{52}$$

for all $\mathbf{p} \in \mathcal{S}$. Both approximations are derived by Taylor expanding the rational function $V_I(\mathbf{p},t) \equiv \kappa(V_{II}(\mathbf{p},t)) \triangleq 1/V_{II}(\mathbf{p},t)$ around the *estimator* $\mathbb{E}\{V_{II}(\mathbf{p},t)|\mathscr{C}(\mathcal{T}_{t-1})\}$, and then taking conditional expectations on the resulting expressions, relative to $\mathscr{C}(\mathcal{T}_{t-1})$. See also our quantitative justification of (51) and (52) below. Then, the proposed order-1 and order-2 MSD approximations of problem (34) are

$$\boxed{\begin{array}{ll}\underset{\mathbf{p}}{\text{maximize}} & \mathsf{T}^E_1(\mathbf{p},t)\\ \text{subject to} & \mathbf{p} \in \mathcal{C}_i(\mathbf{p}^o(t-1))\end{array}} \tag{53}$$

and

$$\boxed{\begin{array}{ll}\underset{\mathbf{p}}{\text{maximize}} & \mathsf{T}^E_2(\mathbf{p},t)\\ \text{subject to} & \mathbf{p} \in \mathcal{C}_i(\mathbf{p}^o(t-1))\end{array},} \tag{54}$$

at relay $i \in \mathbb{N}^+_R$, solved at each $t-1 \in \mathbb{N}^+_{N_T-1}$.

It is straightforward to show that the square on the numerator of (52) can be expanded into a sum of terms of the form $C(m,n) \times |f(\mathbf{p},t)|^m |g(\mathbf{p},t)|^n$, for $(m,n) \in \mathbb{Z}\times\mathbb{Z}$ and some constant $C(m,n)$. Consequently, owing to Lemma 5, the approximate formula (52) (and thus also (51)) may be efficiently computed in closed form at any point $\mathbf{p} \in \mathcal{S}$.

Justification of the MSD: In our context, the MSD approach may be quantitatively justified as follows. By Taylor's Theorem, it is true that $V_I(\mathbf{p},t)$ may be expressed around any $\mathscr{C}(\mathcal{T}_{t-1})$-measurable random variable $Z : \Omega \to \mathbb{R}_{++}$ (a function of the information collected *so far*) in terms of a *Taylor polynomial* and the respective *Taylor remainder* as

$$V_I(\mathbf{p},t) \equiv \mathsf{T}_j(\mathbf{p},t;Z) + \mathsf{R}_j(\mathbf{p},t;Z), \tag{55}$$

for $j \in \{1,2\}$, where

$$\begin{aligned}\mathsf{T}_j(\mathbf{p},t;Z) \triangleq{}& \frac{1}{Z} - \frac{1}{Z^2}(V_{II}(\mathbf{p},t)-Z)\\ &+ \frac{1}{Z^3}(V_{II}(\mathbf{p},t)-Z)^2\,\mathbb{1}_{\{j\equiv 2\}}\end{aligned} \tag{56}$$

and

$$\begin{aligned}\mathsf{R}_j(\mathbf{p},t;Z) &\triangleq V_I(\mathbf{p},t) - \mathsf{T}_j(\mathbf{p},t;Z)\\ &= (-1)^{j+1}(\zeta_j)^{-(j+2)}(V_{II}(\mathbf{p},t)-Z)^{j+1},\end{aligned} \tag{57}$$

for all $(\mathbf{p},t) \in \mathcal{S}\times\mathbb{N}^2_{N_T}$, for some function $\zeta_j : \Omega \to \mathbb{R}_{++}$, such that $\zeta_j \in (Z, V_{II}(\mathbf{p},t))$, whenever $Z \le V_I(\mathbf{p},t)$, and $\zeta_j \in (V_I(\mathbf{p},t), Z)$, otherwise. In (56), the choice of Z is crucial, so that the resulting approximations are as accurate as possible. Therefore, we will be interested in *selecting* the reference point Z in (56) in an operationally meaningful fashion.

Hereafter, *fix* $(\mathbf{p},t) \in \mathcal{S}\times\mathbb{N}^2_{N_T}$, and let $j \equiv 1$. In this case, the remainder of the Taylor expansion takes the form

$$\begin{aligned}&\mathsf{R}_1(\mathbf{p},t;Z)\\ &\equiv \frac{1}{V_{II}(\mathbf{p},t)} - \frac{1}{Z} - \frac{\partial}{\partial x}\left(\frac{1}{x}\right)\bigg|_{x\equiv Z}(V_{II}(\mathbf{p},t)-Z).\end{aligned} \tag{58}$$

It is most reasonable to select Z, so that R_1 is minimized, in an appropriate sense. We observe that R_1 is nothing but the *Bregman divergence* of the *strictly convex* function $\kappa(\cdot) \equiv 1/(\cdot)$ on $(0,\infty)$ [44], evaluated at the pair $(V_{II}(\mathbf{p},t), Z)$. Denote this divergence as $\mathsf{d}_\kappa : \mathbb{R}^2_{++} \to \mathbb{R}$. We have $\mathsf{d}_\kappa(x,y) \ge 0$, for every qualifying pair (x,y). In particular, $\mathsf{R}_1(\mathbf{p},t;Z) \ge 0$, for every feasible choice of Z. Set $\mathbb{E}\{\cdot|\mathscr{C}(\mathcal{T}_{t-1})\} \equiv \mathbb{E}_{t-1}\{\cdot\}$, for brevity. Then, *for every* Z in the space of strictly positive, $\mathscr{C}(\mathcal{T}_{t-1})$-measurable random variables, it follows that

$$\begin{aligned}&\mathbb{E}_{t-1}\{\mathsf{R}_1(\mathbf{p},t;Z)\} - \mathbb{E}_{t-1}\{\mathsf{R}_1(\mathbf{p},t;\mathbb{E}_{t-1}\{V_{II}(\mathbf{p},t)\})\}\\ &= \frac{1}{\mathbb{E}_{t-1}\{V_{II}(\mathbf{p},t)\}} - \frac{1}{Z} + \frac{1}{Z^2}(\mathbb{E}_{t-1}\{V_{II}(\mathbf{p},t)\}-Z)\\ &= \mathsf{d}_\kappa(\mathbb{E}_{t-1}\{V_{II}(\mathbf{p},t)\}, Z) \ge 0,\end{aligned} \tag{59}$$

with equality attained uniquely at $Z^* \equiv \mathbb{E}_{t-1}\{V_{II}(\mathbf{p},t)\}$. In other words, Z^* solves the pointwise optimization problem

$$\underset{z>0}{\text{minimize}} \quad \mathbb{E}_{t-1}\{\mathsf{R}_1(\mathbf{p},t;z)\}. \tag{60}$$

Interestingly, by the IP [40], discussed for another purpose earlier in Section IV-C, it is also true that Z^* solves the policy search, *variational problem*

$$\begin{array}{ll}\underset{z}{\text{minimize}} & \mathbb{E}\{\mathsf{R}_1(\mathbf{p},t;Z)\}\\ \text{subject to} & Z \in \mathbb{R}_{++}\\ & Z \text{ is } \mathscr{C}(\mathcal{T}_{t-1})\text{-measurable}\end{array}. \tag{61}$$

Substituting for Z^* for Z in (56), the resulting *optimal* (in the sense of both (60) and (61)) order-1 Taylor expansion is

$$\begin{aligned}&\mathsf{T}_1(\mathbf{p},t;\mathbb{E}_{t-1}\{V_{II}(\mathbf{p},t)\})\\ &\equiv \frac{2}{\mathbb{E}_{t-1}\{V_{II}(\mathbf{p},t)\}} - \frac{V_{II}(\mathbf{p},t)}{(\mathbb{E}_{t-1}\{V_{II}(\mathbf{p},t)\})^2},\end{aligned} \tag{62}$$

and by taking conditional expectations on both sides, we finally obtain

$$\mathbb{E}_{t-1}\{\mathsf{T}_1(\mathbf{p},t;\mathbb{E}_{t-1}\{V_{II}(\mathbf{p},t)\})\} \equiv \mathsf{T}^E_1(\mathbf{p},t), \tag{63}$$

for every choice of $(\mathbf{p},t) \in \mathcal{S}\times\mathbb{N}^2_{N_T}$.

Turning to the deviation of optimal value of the surrogate (51) from that of (34), it is of course true that

$$\begin{aligned}&\left|\sup_{\mathbf{p}}\mathbb{E}_{t-1}\{V_I(\mathbf{p},t)\} - \sup_{\mathbf{p}}\mathsf{T}^E_1(\mathbf{p},t)\right|\\ &\le \sup_{\mathbf{p}}\left|\mathbb{E}_{t-1}\{V_I(\mathbf{p},t)\} - \mathsf{T}^E_1(\mathbf{p},t)\right|\\ &\equiv \sup_{\mathbf{p}}\inf_{z>0}\mathbb{E}_{t-1}\{\mathsf{R}_1(\mathbf{p},t;z)\}\\ &\equiv \sup_{\mathbf{p}}\mathbb{E}_{t-1}\left\{\frac{1}{V_{II}(\mathbf{p},t)}\right\} - \frac{1}{\mathbb{E}_{t-1}\{V_{II}(\mathbf{p},t)\}}.\end{aligned} \tag{64}$$

This means that, for each $t \in \mathbb{N}^2_{N_T}$, the optimal values of (51) and (34) differ at most by the optimal Taylor reminder, *maximized* over the set of feasible relay positions $\mathcal{C}_i(\mathbf{p}^o(t-1))$, for $i \in \mathbb{N}_R^+$. Observe that our approach *does not* necessarily yield minimization of the worst deviation between optimal values of (51) and (34); this seems to be a much harder problem to solve, and is not considered in this paper.

The case of the proposed order-2 Taylor surrogate (52) is obtained by setting $j \equiv 2$ in (56), and choosing $Z \equiv \mathbb{E}_{t-1}\{V_{II}(\mathbf{p},t)\}$, as well. The justification of (52) is heuristic. As already shown in, for instance, (64), by convexity of the ratio function κ, Jensen's Inequality directly implies that the objective of (53), $\mathsf{T}_1^E(\mathbf{p},t)$, is always *lower than or equal* that of the original program (34). But observe that Jensen's Inequality and convexity of the square also implies that

$$\begin{aligned}\mathsf{T}_2^E(\mathbf{p},t) &\equiv \frac{\mathbb{E}\left\{(V_{II}(\mathbf{p},t))^2 \middle| \mathscr{C}(\mathcal{T}_{t-1})\right\}}{(\mathbb{E}\{V_{II}(\mathbf{p},t)|\mathscr{C}(\mathcal{T}_{t-1})\})^3} \\ &\geq \frac{(\mathbb{E}\{V_{II}(\mathbf{p},t)|\mathscr{C}(\mathcal{T}_{t-1})\})^2}{(\mathbb{E}\{V_{II}(\mathbf{p},t)|\mathscr{C}(\mathcal{T}_{t-1})\})^3} \equiv \mathsf{T}_1^E(\mathbf{p},t), \quad (67)\end{aligned}$$

where conditioning is, of course, on identical information. This motivates the consideration of the order-2 Taylor approximation $\mathsf{T}_2^E(\mathbf{p},t)$ as a potential refinement of $\mathsf{T}_1^E(\mathbf{p},t)$. As we will see in Section (VII), $\mathsf{T}_2^E(\mathbf{p},t)$ achieves *faster convergence* to steady state than that achieved by $\mathsf{T}_1^E(\mathbf{p},t)$, *empirically* justifying program (52) as an effective heuristic approximation (34). Note, however, that the objective of (53) might still be desirable in practice, since it is somewhat easier to compute.

Remark 6. The reader will notice that we have not considered Taylor expansions of orders greater than two. This is due to the fact that, as (57) demonstrates (this formula holds for $j > 2$, as well), there is no indication that a higher order expansion would diminish the approximation error. On the contrary, we observe the presence of higher order exponents on both numerator and denominator of the remainder formula. Due to such form of the remainder, higher order expansions are potentially highly unstable; this is a common issue of locally convergent Taylor approximations. In regard to the problem considered herein, this unstable behavior has indeed been observed via simulations during the development of this paper. Therefore, the discussion of Taylor expansions of orders greater than two has been deliberately omitted. ■

VI. Short Discussion: Computational Complexity

At this point, it is important to note that, for *each* $\mathbf{p} \in \mathcal{S}$, computation of the conditional mean and covariance in (35) of Lemma 5 require execution of matrix operations, which are of *expanding dimension in* $t \in \mathbb{N}_{N_T}^2$. The reader may also observe that the inversion of $\mathbf{\Sigma}_{1:t-1}$ (of size $2R(t-1)$) constitutes the computationally dominant operation in the long formulas of Lemma 5. The computational complexity of this matrix inversion, taking place at each time slot $t-1 \in \mathbb{N}_{N_T-1}^+$, is, in general, of the order of $\mathcal{O}\left(R^3t^3\right)$ elementary operations. Fortunately though, we may reduce the complexity of the aforementioned matrix inversion to the order of $\mathcal{O}\left(R^3t^2\right)$. By construction, $\mathbf{\Sigma}_{1:t-1}$ may be expressed as

$$\mathbf{\Sigma}_{1:t-1} \equiv \begin{bmatrix} \mathbf{\Sigma}_{1:t-2} & \mathbf{\Sigma}_{1:t-2}^c \\ (\mathbf{\Sigma}_{1:t-2}^c)^{\boldsymbol{T}} & \mathbf{\Sigma}(t-1,t-1) \end{bmatrix}, \quad (68)$$

where $\mathbf{\Sigma}_{1:t-2}^c \in \mathbb{R}^{2R(t-2)\times 2R}$ is defined as

$$\mathbf{\Sigma}_{1:t-2}^c \triangleq [\mathbf{\Sigma}(1,t-1) \ \ldots \ \mathbf{\Sigma}(t-2,t-1)]^{\boldsymbol{T}}. \quad (69)$$

Invoking the Matrix Inversion Lemma, we obtain the *recursive* expression as defined in (65) and (66) (bottom of page), where S_{t-1} is the respective Schur complement. From (65) and (66), it can be easily verified that the most computationally demanding operation involved is $\mathbf{\Sigma}_{1:t-2}^{-1}\mathbf{\Sigma}_{1:t-2}^c$, of order $\mathcal{O}\left(R^3t^2\right)$. Since the inversion of S_{t-1} is of the order of $\mathcal{O}\left(R^3\right)$, we arrive at a total reduced complexity of $\mathcal{O}\left(R^3t^2\right)$ elementary operations of the recursive scheme presented above.

The achieved reduction in complexity is important. In most scenarios, R, the number of relays, will be relatively small and fixed for the whole operation of the system, whereas t, the time slot index, might generally take large values, since it is common for the operational horizon of the system, N_T, to be large. Additionally, the reader may readily observe that the aforementioned covariance matrix is independent of the position at which the channel is predicted, $\mathbf{p}$. As a result, its inversion may be performed *just once in each time slot*, for all evaluations of the mean and covariance of the Gaussian density in (35), for all distinct choices of $\mathbf{p}$. Consequently, if the total number of such evaluations is $P \in \mathbb{N}^+$, and recalling that the complexity for a (square) matrix-vector multiplication is quadratic in the dimension of the quantities involved, then, *at worst*, the total computational complexity *for channel prediction* is of the order of $\mathcal{O}\left(PR^2t^2 + R^3t^2\right)$, *at each* $t-1 \in \mathbb{N}_{N_T-1}^+$, with worst case complexity of an order of $\mathcal{O}\left(PR^2N_T^2 + R^3N_T^2\right)$.

Since, for each relay $i \in \mathbb{N}_R^+$, the feasible set $\mathcal{C}_i$ is assumed to be finite, the analysis above characterizes the complexity for solving either of all four optimization surrogates presented in Section V. For any set $\mathcal{A}$, let $|\mathcal{A}|$ denote its cardinality. Assuming that computation of the square root of a positive semidefinite two-by-two matrix is a fixed-complexity operation, then, for every pair $(i,t) \in \mathbb{N}_R^+ \times \mathbb{N}_{N_T}^2$, the complexities for each of the four surrogates are:

- SAA: $\mathcal{O}\left(|\mathcal{C}_i| S + |\mathcal{C}_i| R^2t^2 + R^3t^2\right)$.
- GHQ: $\mathcal{O}\left(|\mathcal{C}_i| Q^2 + |\mathcal{C}_i| R^2t^2 + R^3t^2\right)$.
- MSD (order-1 *and* order-2): $\mathcal{O}\left(|\mathcal{C}_i| R^2t^2 + R^3t^2\right)$.

Although in the case of the GHQ the dependence on the number of quadrature points, Q, is quadratic, compared to

$$\mathbf{\Sigma}_{1:t-1}^{-1} = \begin{bmatrix} \mathbf{\Sigma}_{1:t-2}^{-1} + \mathbf{\Sigma}_{1:t-2}^{-1}\mathbf{\Sigma}_{1:t-2}^c\mathsf{S}_{t-1}^{-1}(\mathbf{\Sigma}_{1:t-2}^c)^{\boldsymbol{T}}\mathbf{\Sigma}_{1:t-2}^{-1} & -\mathbf{\Sigma}_{1:t-2}^{-1}\mathbf{\Sigma}_{1:t-2}^c\mathsf{S}_{t-1}^{-1} \\ -\mathsf{S}_{t-1}^{-1}(\mathbf{\Sigma}_{1:t-2}^c)^{\boldsymbol{T}}\mathbf{\Sigma}_{1:t-2}^{-1} & \mathsf{S}_{t-1}^{-1} \end{bmatrix} \quad (65)$$

$$\mathsf{S}_{t-1} \triangleq \mathbf{\Sigma}(t-1,t-1) - (\mathbf{\Sigma}_{1:t-2}^c)^{\boldsymbol{T}}\mathbf{\Sigma}_{1:t-2}^{-1}\mathbf{\Sigma}_{1:t-2}^c \in \mathbb{S}_{++}^{2R} \quad (66)$$

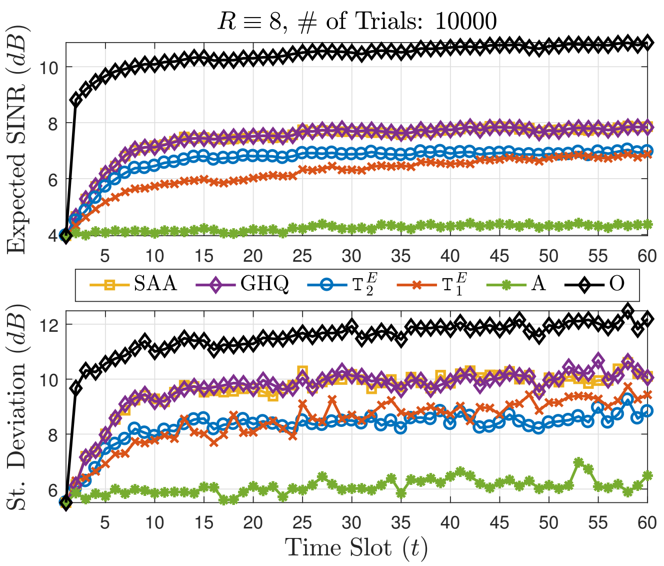

Figure 4: Comparison of proposed relay motion policies. In the figure, "A" stands for "Agnostic" and "O" stands for "Oracle".

a linear dependence on the sample size S for the case of the SAA, in practice it *always* the case that $Q \ll S$. See also our numerical results in Section VII below. This renders the complexity of the SAA and the GHQ totally comparable. In fact, sometimes the GHQ is more numerically efficient than SAA, for achieving the same approximation accuracy.

In terms of practical feasibility, and especially for systems with a long operational horizon (N_T), implementation issues due to potentially high complexity can be mitigated in various ways. Examples include the exploitation of a strong network node, or fusion center (for centralized systems), or some dedicated, distributed cloud computing service. Additionally, in order to deal with matrix operations of expanding dimensions, *sliding-window channel prediction* may be implemented, where, at each time slot, each relay conditions on past channel observations only up to a *certain fixed lag*. Such an approach is expected to work very well and for a relatively small window size, due to the exponentially decaying structure of the temporal correlation component of the channel model.

VII. Numerical Simulations

In this section, we present synthetic numerical simulations, which confirm that the proposed 2-stage approach works, and yields substantially improved beamforming performance. All experiments were conducted on an imaginary square terrain of dimensions 30×30 squared units of length, with $\mathcal{W} \equiv [0, 30]^2$, uniformly divided into $30 \times 30 \equiv 900$ square regions. The source and destination are fixed at $\mathbf{p}_S \equiv [15\,0]^{\boldsymbol{T}}$ and $\mathbf{p}_D \equiv [15\,30]^{\boldsymbol{T}}$. The beamforming horizon is chosen as $T \equiv 60$ and the number of relays is fixed at $R \equiv 8$. The wavelength is chosen as $\lambda \equiv 0.125$, matching a carrier frequency of $2.4\,GHz$. The various channel parameters are set as $\ell \equiv 3$, $\rho \equiv 20$, $\sigma_\xi^2 \equiv 20$, $\eta^2 \equiv 50$, $\beta \equiv 10$, $\gamma \equiv 5$ and $\delta \equiv 1$. The variances of noises at the relays and destination are fixed as $\sigma^2 \equiv \sigma_D^2 \equiv 1$. Lastly, both the transmission power of the source and the *total* power budget of the relays are chosen as $P_0 \equiv P_c \equiv 25$ ($\approx 14dB$) units of power.

The relays are restricted to the rectangular region $\mathcal{S} \equiv [0, 30] \times [12, 18]$. At each time instant, each relay is allowed to move inside a 9-region area, centered at its current position, thus defining its feasible set $\mathcal{C}_i(\cdot)$, $i \in \mathbb{N}_R^+$. Basic collision and out-of-bounds control was also considered and implemented.

In order to assess the effectiveness of our proposed approach, we compare all four proposed surrogates to the base problem (34), proposed in Section V, against the case of an *agnostic*, *purely randomized* relay control policy; in this case, at each time slot, each relay moves randomly to a new available position, without exploiting observed CSI. For reference, we also consider the performance of an *oracle* control policy at the relays, where, at each time slot $t-1 \in \mathbb{N}_{N_T-1}^+$, relay $i \in \mathbb{N}_R^+$ updates its position by *noncausally* looking into the future and choosing the position $\mathbf{p}_i(t)$, which maximizes directly the quantity $V_I(\mathbf{p}_i(t), t)$, over $\mathcal{C}_i(\mathbf{p}_i(t-1))$. In other words, the oracle control policy is implemented by assuming access to the CSI at every possible position of each relay at time slot t, *while being at time slot* $t-1$. Of course, the oracle control policy is not implementable in reality, but only in simulation, and corresponds to an *unachievable performance upper bound*, useful only for the purpose of evaluating the quality of a particular implementable policy, that is, for benchmarking. Of course, comparison of all controlled systems is made under exactly the same communication environment.

Fig. 4 shows the expectation and standard deviation of the achieved QoS for all controlled systems, approximated by executing 10000 trials of the whole experiment. The sample size and quadrature resolution of the SAA and GHQ surrogates are set to $S \equiv 1000$ and $Q = 8$, respectively. Also, in the case of the SAA, *only one* sample has been generated for all relays, and for all times slots. As seen in the figure, there is a clear advantage in exploiting strategically designed relay motion control. Whereas the agnostic system maintains an average SINR of about $4\,dB$ at all times, the SAA and GHQ surrogates are clearly superior, exhibiting an increasing trend in the achieved SINR, with a gap starting from about $0.5\,dB$ at time slot $t \equiv 2$, up to $3.5\,dB$ at time slots $t \equiv 12, 13, \ldots, 60$. More specifically, assuming that the GHQ surrogate has reached a (quasi) steady state when $t \in [12, 60]$, the *average* performance gap in steady state is $3.4116\,dB$, which translates into *an average improvement of about* 80% *on the average network SINR at steady state*, compared to the agnostic policy. As seen in Fig. 4, the performance of the SAA surrogate is almost identical to that of GHQ surrogate; thus it is not further discussed.

The order-2 MSD surrogate (problem (54)) comes second to SAA and GHQ, with always lower average SINR of a relative gap of approximately $1\,dB$, and which also exhibits a similar increasing trend. The order-1 MSD surrogate comes last and, albeit at a slow rate, it seems to converge to (and possibly even surpass) the QoS achieved by the order-2 MSD surrogate. Still, the fact that the order-2 MSD surrogate converges faster to steady state than the order-1 MSD one confirms our expectation that the former is a somewhat refined version of the latter. Given that both MSD heuristics are super computationally efficient, their performance indicates that they are excellent cheap alternatives to SAA and GHQ.

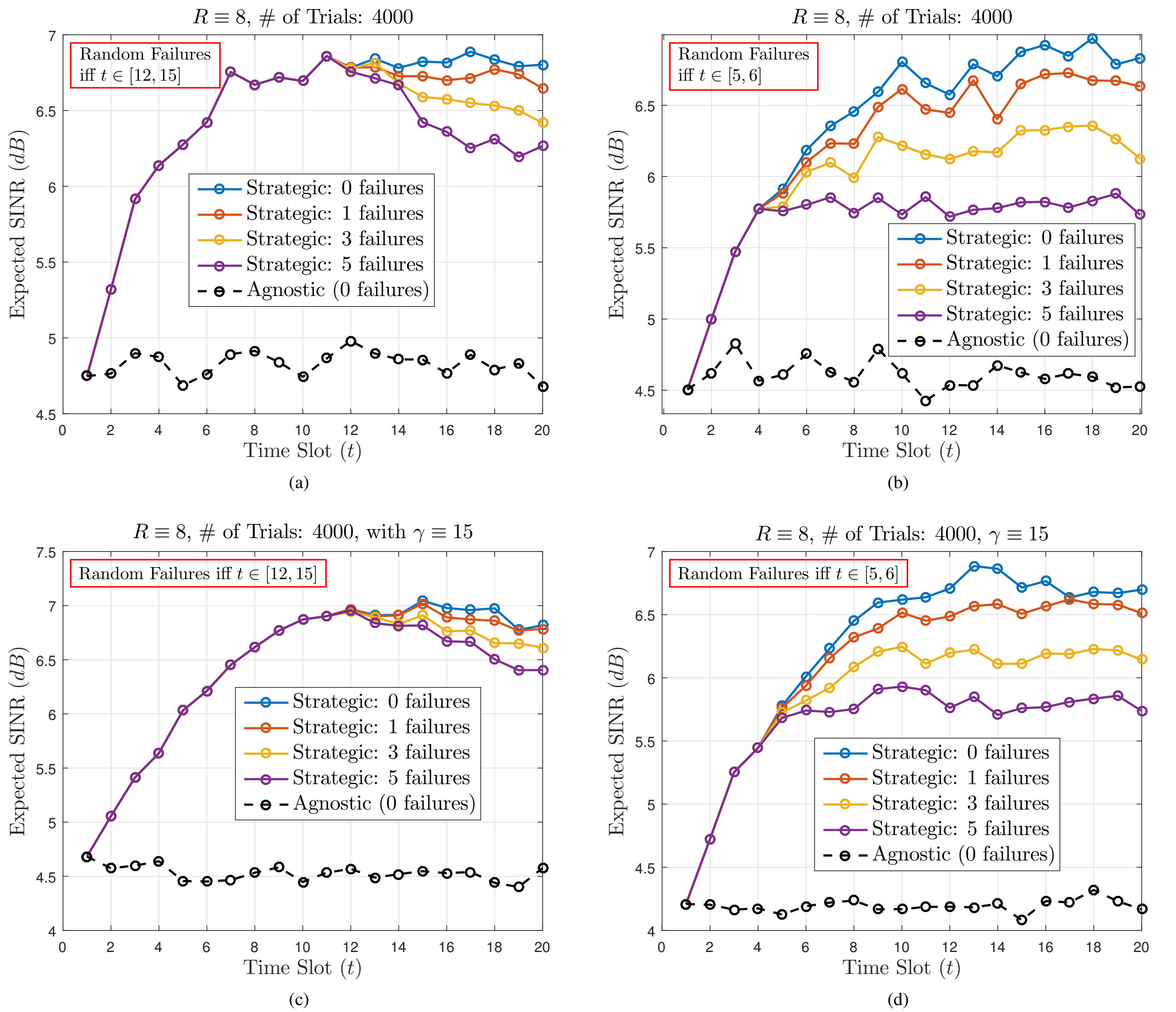

Figure 5: Performance of the proposed spatially controlled system, at the presence of motion failures.

Consequently, it is experimentally verified that, although the proposed stochastic programming approach is myopic, the resulting system performance is not, and this depends on the fact that the channel exhibits non trivial temporal statistical interactions. We should also comment on the standard deviation of all systems, which, from Fig. 4, seems somewhat high, relative to the range of the respective average SINR. This is *exclusively* due to the wild variations of the channel, which, in turn, are due to the effects of shadowing and multipath fading. This is reasonable, since, when the channel is not *actually* in deep fade at time t (an event which might happen with positive probability), the relays, at time $t-1$, are predictively steered to locations, which, on average, incur higher network QoS. As clearly shown in Fig. 4, for all systems under study, an increase in system performance also implies a proportional increase in the respective standard deviation.

Next, we experimentally evaluate system performance at the presence of *random motion failures* in the network. Hereafter, we work with the $2nd$ order heuristic (54) (an average-quality solution), and set $T \equiv 20$. Random motion failures are modeled by choosing, at each trial, a *random sample* of a fixed number of relays and a *random time* when the failures occur. At that time, the selected relays just stop moving, but they continue to beamform, from the position each of them visited last. Two cases are considered; first, motion failures happen if and only if $t \in [12, 15]$ (Figs. 5a and 5c), whereas, in the second case, $t \in [5, 6]$ (Figs. 5b and 5d). In both cases, zero, one, three and five relays (chosen at random, at each trial) stop moving. Two cases for γ are considered, $\gamma \equiv 5$ (Figs. 5a and 5b) and $\gamma \equiv 15$ (Figs. 5c and 5d).

Fig. 5a clearly demonstrates that a larger number of motion failures induces a proportional, relatively (depending on γ) slight decrease in performance; this decrease, though, is smoothly evolving, and is not abrupt. This behavior is more pronounced in Fig. 5c, where the correlation time parameter γ has been increased to 15. We readily observe that, in this case, over the same horizon, the operation of the system is smoother, and decrease in performance, as well as its slope, are significantly smaller than those in Fig. 5a, for all cases of motion failures. Now, in Figs. 5b and 5d, when motion failures happen early, well before the network QoS converges to its maximal value, we observe that, although some relays might stop moving at some point, the achieved expected network QoS continues exhibiting its usual increasing trend. Of course, the performance of the system converges to values strictly proportional to the number of failures in each of the cases considered. This means that the relays which continue moving contribute positively to increasing network QoS.

VIII. Conclusions

We have considered the problem of enhancing QoS in time slotted relay beamforming networks with one source/destination, via stochastic relay motion control. Modeling the wireless channel as a spatiotemporal stochastic field, we proposed a novel 2-stage stochastic programming approach for jointly specifying beamforming weights and relay positions, such that the expected network QoS is maximized, based on causal CSI and under a total relay power constraint. We have shown that this problem can be effectively approximated by a set of simple, two dimensional and theoretically justified surrogate subproblems, which can be distributively solved, one at each relay. The proposed surrogates, each which rely on the SAA, the GHQ, and the MSD, respectively, present a nice trade-off between performance and numerical efficiency. Our simulations have revealed several important properties and confirmed the success of the proposed approach, which results in motion policies yielding substantial performance gains, reportedly an improvement of about 80% on the average network QoS at steady state, as compared to agnostic, randomized relay motion.

Appendix: Proofs

Proof of Lemma 2

In the following, we construct $\boldsymbol{\Sigma}$ *incrementally*. Initially, consider the matrix $\widetilde{\boldsymbol{\Sigma}} \in \mathbb{S}^{RN_T}$ defined as

$$\widetilde{\boldsymbol{\Sigma}} \triangleq \begin{bmatrix} \widetilde{\boldsymbol{\Sigma}}(1,1) & \dots & \widetilde{\boldsymbol{\Sigma}}(1,N_T) \\ \vdots & \ddots & \vdots \\ \widetilde{\boldsymbol{\Sigma}}(N_T,1) & \cdots & \widetilde{\boldsymbol{\Sigma}}(N_T,N_T) \end{bmatrix}, \tag{70}$$

where, for each $(k,l) \in \mathbb{N}^+_{N_T} \times \mathbb{N}^+_{N_T}$, $\widetilde{\boldsymbol{\Sigma}}(k,l) \in \mathbb{S}^R$, with

$$\widetilde{\boldsymbol{\Sigma}}(k,l)(i,j) \triangleq \widetilde{\boldsymbol{\Sigma}}\left(\mathbf{p}_i(k), \mathbf{p}_j(l)\right) \triangleq \eta^2 e^{-\frac{\left\|\mathbf{p}_i(k)-\mathbf{p}_j(l)\right\|_2}{\beta}}, \tag{71}$$

for all $(i,j) \in \mathbb{N}^+_R \times \mathbb{N}^+_R$. By construction, $\widetilde{\boldsymbol{\Sigma}} \in \mathbb{S}^{RN_T}_+$, because the well known *exponential kernel* $\widetilde{\boldsymbol{\Sigma}} : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}_{++}$ defined above is positive (semi)definite.

Next, define the positive definite matrix

$$\mathbf{K} \triangleq \begin{bmatrix} 1 & \kappa \\ \kappa & 1 \end{bmatrix}, \text{ with } \kappa \triangleq \exp\left(-\frac{\|\mathbf{p}_S - \mathbf{p}_D\|_2}{\delta}\right) < 1 \tag{72}$$

and consider the *Tracy-Singh* type of product of $\mathbf{K}$ and $\widetilde{\boldsymbol{\Sigma}}$

$$\widetilde{\boldsymbol{\Sigma}}_{\mathbf{K}} \triangleq \mathbf{K} \circ \widetilde{\boldsymbol{\Sigma}} \in \mathbb{S}^{2RN_T}$$

$$\triangleq \begin{bmatrix} \mathbf{K} \otimes \widetilde{\boldsymbol{\Sigma}}(1,1) & \dots & \mathbf{K} \otimes \widetilde{\boldsymbol{\Sigma}}(1,N_T) \\ \vdots & \ddots & \vdots \\ \mathbf{K} \otimes \widetilde{\boldsymbol{\Sigma}}(N_T,1) & \cdots & \mathbf{K} \otimes \widetilde{\boldsymbol{\Sigma}}(N_T,N_T) \end{bmatrix}, \tag{73}$$

where "$\otimes$" denotes the operator of the Kronecker product. Then, for each $(k,l) \in \mathbb{N}^+_{N_T} \times \mathbb{N}^+_{N_T}$, $\mathbf{K} \otimes \widetilde{\boldsymbol{\Sigma}}(k,l) \in \mathbb{S}^{2R}$. It is easy to show that $\widetilde{\boldsymbol{\Sigma}}_{\mathbf{K}} \in \mathbb{S}^{2R}_+$. First, via a simple inductive argument, it follows that, for compatible matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$, $(\mathbf{AB}) \circ (\mathbf{CD}) \equiv (\mathbf{A} \circ \mathbf{C})(\mathbf{B} \circ \mathbf{D})$. Also, for compatible $\mathbf{A}, \mathbf{B}$, it is true that $(\mathbf{A} \circ \mathbf{B})^{\boldsymbol{T}} \equiv \mathbf{A}^{\boldsymbol{T}} \circ \mathbf{B}^{\boldsymbol{T}}$. Since $\mathbf{K}$ and $\widetilde{\boldsymbol{\Sigma}}$ are symmetric, consider their spectral decompositions $\mathbf{K} \equiv \mathbf{U_K \Lambda_K U_K^{\boldsymbol{T}}}$ and $\widetilde{\boldsymbol{\Sigma}} \equiv \mathbf{U}_{\widetilde{\boldsymbol{\Sigma}}} \boldsymbol{\Lambda}_{\widetilde{\boldsymbol{\Sigma}}} \mathbf{U}_{\widetilde{\boldsymbol{\Sigma}}}^{\boldsymbol{T}}$. We may then write

$$\begin{aligned} \widetilde{\boldsymbol{\Sigma}}_{\mathbf{K}} &\equiv \left(\mathbf{U_K \Lambda_K U_K^{\boldsymbol{T}}}\right) \circ \left(\mathbf{U}_{\widetilde{\boldsymbol{\Sigma}}} \boldsymbol{\Lambda}_{\widetilde{\boldsymbol{\Sigma}}} \mathbf{U}_{\widetilde{\boldsymbol{\Sigma}}}^{\boldsymbol{T}}\right) \\ &\equiv \left(\mathbf{U_K} \circ \mathbf{U}_{\widetilde{\boldsymbol{\Sigma}}}\right)\left(\boldsymbol{\Lambda}_{\mathbf{K}} \circ \boldsymbol{\Lambda}_{\widetilde{\boldsymbol{\Sigma}}}\right)\left(\mathbf{U}_{\mathbf{K}}^{\boldsymbol{T}} \circ \mathbf{U}_{\widetilde{\boldsymbol{\Sigma}}}^{\boldsymbol{T}}\right) \\ &\equiv \left(\mathbf{U_K} \circ \mathbf{U}_{\widetilde{\boldsymbol{\Sigma}}}\right)\left(\boldsymbol{\Lambda}_{\mathbf{K}} \circ \boldsymbol{\Lambda}_{\widetilde{\boldsymbol{\Sigma}}}\right)\left(\mathbf{U_K} \circ \mathbf{U}_{\widetilde{\boldsymbol{\Sigma}}}\right)^{\boldsymbol{T}}, \end{aligned} \tag{74}$$

where $\left(\mathbf{U_K} \circ \mathbf{U}_{\widetilde{\boldsymbol{\Sigma}}}\right)\left(\mathbf{U}_{\mathbf{K}}^{\boldsymbol{T}} \circ \mathbf{U}_{\widetilde{\boldsymbol{\Sigma}}}^{\boldsymbol{T}}\right) \equiv \left(\mathbf{U_K U_K^{\boldsymbol{T}}}\right) \circ \left(\mathbf{U}_{\widetilde{\boldsymbol{\Sigma}}} \mathbf{U}_{\widetilde{\boldsymbol{\Sigma}}}^{\boldsymbol{T}}\right) \equiv \mathbf{I}_2 \circ \mathbf{I}_{RN_T} \equiv \mathbf{I}_{2RN_T}$, and where the matrix $\boldsymbol{\Lambda}_{\mathbf{K}} \circ \boldsymbol{\Lambda}_{\widetilde{\boldsymbol{\Sigma}}}$ is easily shown to be diagonal and with nonnegative elements. Thus, since (74) constitutes a valid spectral decomposition for $\widetilde{\boldsymbol{\Sigma}}_{\mathbf{K}}$, it follows that $\widetilde{\boldsymbol{\Sigma}}_{\mathbf{K}} \in \mathbb{S}^{2RN_T}_+$.

As a last step, let $\mathbf{E} \in \mathbb{S}^{N_T}$, such that $\mathbf{E}(k,l) \triangleq \exp(-|k-l|/\gamma)$, for all $(k,l) \in \mathbb{N}^+_{N_T} \times \mathbb{N}^+_{N_T}$. Again, $\mathbf{E}$ is positive semidefinite, because the well known *Laplacian kernel* is positive (semi)definite. Consider the matrix $\widetilde{\boldsymbol{\Sigma}}_{\mathbf{E}} \triangleq (\mathbf{E} \otimes \mathbf{1}_{2R\times 2R}) \odot \widetilde{\boldsymbol{\Sigma}}_{\mathbf{K}} \in \mathbb{S}^{2RN_T}$, where "$\odot$" denotes the operator of the Schur-Hadamard product. Of course, since the matrix $\mathbf{1}_{2R\times 2R}$ is rank-1 and positive semidefinite, $\mathbf{E} \otimes \mathbf{1}_{2R\times 2R}$ will be positive semidefinite as well. Consequently, by the Schur Product Theorem, $\widetilde{\boldsymbol{\Sigma}}_{\mathbf{E}}$ will also be positive semidefinite. Finally, observe that $\boldsymbol{\Sigma} \equiv \widetilde{\boldsymbol{\Sigma}}_{\mathbf{E}} + \sigma_\xi^2 \mathbf{I}_{2RN_T}$, implying that $\boldsymbol{\Sigma} \in \mathbb{S}^{2RN_T}_{++}$, whenever $\sigma_\xi^2 \neq 0$. Our claims follow. ■

Proof of Lemma 5

In the notation of the statement of the lemma, the joint conditional distribution of $[F(\mathbf{p},t)\ G(\mathbf{p},t)]^{\boldsymbol{T}}$ relative to the σ-algebra $\mathscr{C}(\mathcal{T}_{t-1})$ can be shown to be Gaussian with mean $\boldsymbol{\mu}^{F,G}_{t|t-1}(\mathbf{p})$ and covariance $\boldsymbol{\Sigma}^{F,G}_{t|t-1}(\mathbf{p})$, for all $(\mathbf{p},t) \in \mathcal{S} \times \mathbb{N}^2_{N_T}$. It is then a typical exercise (possibly somewhat tedious though) to show that the functions $\boldsymbol{\mu}^{F,G}_{t|t-1}$ and $\boldsymbol{\Sigma}^{F,G}_{t|t-1}$ are of the form asserted in the statement of the lemma. Regarding the proof for (40), observe that we can write

$$\begin{aligned} &\mathbb{E}\left\{|f(\mathbf{p},t)|^m |g(\mathbf{p},t)|^n \middle| \mathscr{C}(\mathcal{T}_{t-1})\right\} \\ &\equiv 10^{(m+n)\rho/20} \\ &\times \mathbb{E}\left\{\exp\left(\frac{\log(10)}{20}[m\,n]\left[F(\mathbf{p},t)\ G(\mathbf{p},t)\right]^{\boldsymbol{T}}\right)\middle| \mathscr{C}(\mathcal{T}_{t-1})\right\}, \end{aligned} \tag{75}$$

with the quantity on the RHS being nothing else than the conditional moment generating function of the conditionally jointly Gaussian random vector $[F(\mathbf{p},t)\ G(\mathbf{p},t)]^{\boldsymbol{T}}$ at each $\mathbf{p}$ and t, evaluated at the point $(\log(10)/20)[m\,n]^{\boldsymbol{T}}$, for any choice of $(m,n) \in \mathbb{Z} \times \mathbb{Z}$. Recalling the special form of the moment generating function for Gaussian random vectors, the result readily follows. ■

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